

2.5 Limits Involving Infinity; Asymptotes of Graphs

Sometimes limits don't always go to a single number. Sometimes they may go to infinity or negative infinity. You do not need to graph every function. You can utilize tables. Let's look at some examples:

EXAMPLE: Use the table below for $f(x) = \frac{5}{x+3}$ and find: $\lim_{x \rightarrow -3^+} f(x)$, $\lim_{x \rightarrow -3^-} f(x)$, $\lim_{x \rightarrow -3} f(x)$

x	-3.5	-3.1	-3.01	-3.001	-3	-2.999	-2.99	-2.9	-2.5	
f(x)	-10	-50	-500	-5000	undef.	5000	500	50	10	

a.) $\lim_{x \rightarrow -3^-} f(x)$

b.) $\lim_{x \rightarrow -3^+} f(x)$

c.) $\lim_{x \rightarrow -3} f(x)$

EXAMPLE: Find $\lim_{x \rightarrow 1^+} \frac{2+x}{1-x}$

EXAMPLE: Find $\lim_{x \rightarrow 0^-} x^2 - \frac{1}{x}$

EXAMPLE: Find $\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{-2}{\cos x}$

EXAMPLE: Find $\lim_{x \rightarrow 4^-} \frac{x^2}{x^2 + 16}$

EXAMPLE: Find $\lim_{x \rightarrow -\frac{1}{2}^+} \frac{6x^2 + x - 1}{4x^2 - 4x - 3}$

Limits at Infinity

$\lim_{x \rightarrow \infty} \frac{c}{x^n} = 0$, where $n > 0$. (As the bottom increases, the whole fraction decreases.)

For these problems, divide the top and bottom by the highest power of x in the denominator.

EXAMPLE: Find the limit: $\lim_{x \rightarrow \infty} \frac{3x + 2}{x - 4}$

EXAMPLE: Find the limit: $\lim_{x \rightarrow \infty} \frac{3 - 2x}{3x^3 - 1}$

EXAMPLE: Find the limit: $\lim_{x \rightarrow \infty} \frac{3 - 2x^2}{3x - 1}$

EXAMPLE: Find the horizontal asymptote: $y = \frac{5x^{\frac{3}{2}}}{4x^{\frac{3}{2}} + 1}$.

EXAMPLE: Find the limit: $\lim_{x \rightarrow \infty} \frac{5x^{\frac{3}{2}}}{4\sqrt{x} + 1}$

EXAMPLE: Find the limit: $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}}$

$$x = x$$

$$x^2 = x^2.$$

$$x = \pm\sqrt{x^2}.$$

If the limit is going to positive infinity, use the definition $x = \sqrt{x^2}$.

If the limit is going to negative infinity, use $x = -\sqrt{x^2}$.

$$\text{Let } x = \sqrt{x^2}.$$

EXAMPLE: Find the limit: $\lim_{x \rightarrow -\infty} \frac{-3x+1}{\sqrt{x^2+x}}$

Special Limits with Sine and Cosine

$$\lim_{x \rightarrow \infty} \sin x = DNE \quad \lim_{x \rightarrow \infty} \cos x = DNE \quad \lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0 \quad \lim_{x \rightarrow \infty} \frac{\cos x}{x} = 0$$

EXAMPLE: Find the limit: $\lim_{x \rightarrow \infty} \frac{x - \cos x}{x}$

EXAMPLE: Find the limit: $\lim_{x \rightarrow \infty} \cos\left(\frac{1}{x}\right)$

EXAMPLE: Find the limit: $\lim_{x \rightarrow \infty} \frac{4}{3x - \sin x}$