3.2 The Derivative as a Function

Derivative of a function for any value x

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

EXAMPLE: Find the derivative of $f(x) = 1 - x^2$ by the limit process. Then use your answer to find: $f'(-3)$, $f'(0)$, and $f'\left(\frac{1}{2}\right)$.

EXAMPLE: Find the derivative of $f(x) = -5$ by the limit process. Then use your answer to find: $f'(-100)$

EXAMPLE: Find the derivative of $f(x) = \sqrt{x} + 6$ by the limit process.

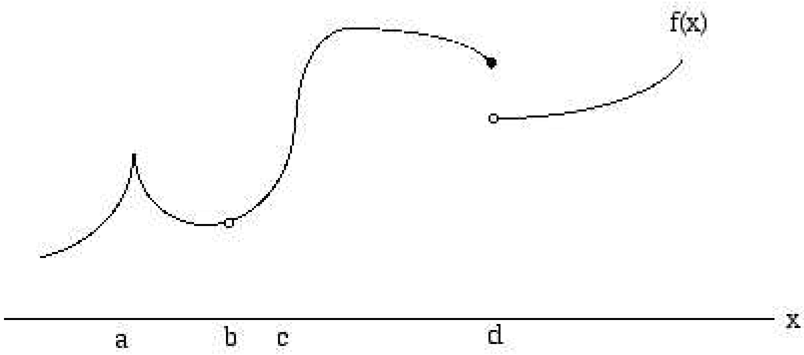
EXAMPLE: Find the derivative of $f(x)=\dfrac{4}{\sqrt{x}}$ by the limit process.

EXAMPLE: Find the derivative of $f(x) = \dfrac{1}{x^2}$ by the limit process.

EXAMPLE: Find the value of $\left.\frac{dy}{dx}\right|_{x=3}$ if $f(x) = x^3 + 2$. Then use it to find the equation of the tangent line at this value of x.

When does a derivative *not* have a derivative at a point?

Because we find derivatives using limits, what if the limit at a certain point does not exist? This means there is no derivative at that point. In the graph below, there are four points where the derivative does not exist.



At point a: There is a cusp (or corner) so the derivative does not exist at a.

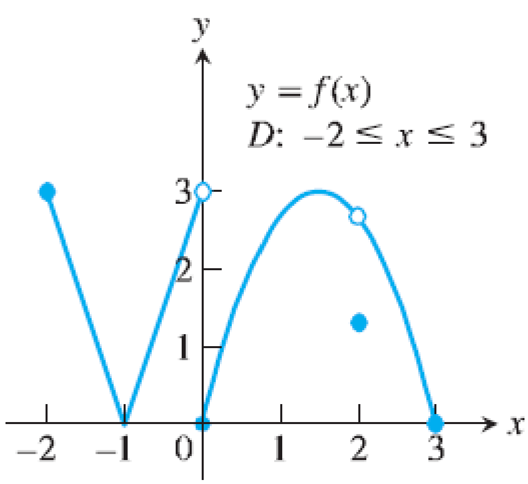
At point b: There is a hole in the graph, so the derivative does not exist at b.

At point c: There is a vertical tangent here, so the derivative does not exist at c.

At point d: There is a discontinuity at d (gap in graph), so the derivative does not exist at d.

EXAMPLE: Find $f'(1)$ if possible given $f(x) = \begin{cases} 2 & \text{if } x \le 1 \\ 2x & \text{if } x > 1 \end{cases}$.

EXAMPLE: The figure below shows a graph of a function over the closed interval [-2, 3]. At what domain points does the graph appear to be: a.) differentiable b.) continuous but not differentiable? c.) neither continuous or differentiable?



a.)

b.)

c.)