3.4 The Derivative as Rates of Change

In an earlier section we introduced average and instantaneous rates of change. In this section we will further discuss other applications in which derivatives would model the rates at which change. In particular we will discuss position, velocity, and acceleration. The table below will illustrate the relationship between these.

Notation	Meaning
$s(t)$	Position at time t
$s'(t) = v(t)$	The derivative of position is velocity
$v'(t) = a(t)$	The derivative of velocity is acceleration

EXAMPLE: Let $s(t) = 4t^2 - t + 9$ be a position function. Find the velocity and acceleration functions.

EXAMPLE: An object is thrown vertically upward whose height is given by the function: $s(t) = -16t^2 + 100$ where t is in seconds and s is in feet. Find the velocity of the object after 3 seconds.

EXAMPLE: Given $s(t) = \frac{t^4}{4} - t^3 + t^2$ on the interval [0, 3] where s is in meters and t is in seconds.

a.) Find the body's displacement and average velocity for the given time interval.

b.) Find the body's speed and acceleration at the endpoints of the interval.

c.) When, if ever, during the interval does the body change direction?

EXAMPLE: At time $t \geq 0$, the velocity of a body moving along the s-axis is $v = t^2 - 4t + 3$.

a.) Find the body's acceleration each time the velocity is zero.

b.) When is the body moving forward? Backward?

c.) When is the body's velocity increasing? Decreasing?

EXAMPLE: A dynamite blast blows a heavy rock straight up with a launch velocity of 160 feet per second (109 mph). It reaches a height of $s(t) = 160t - 16t^2$ after t seconds.

a.) How high does the rock go?

b.) What is the velocity of the rock when it is 256 ft above the ground on the way up? On the way down?

c.) What is the acceleration of the rock at any time t during its flight (after the blast)?

d.) When does the rock hit the ground again?