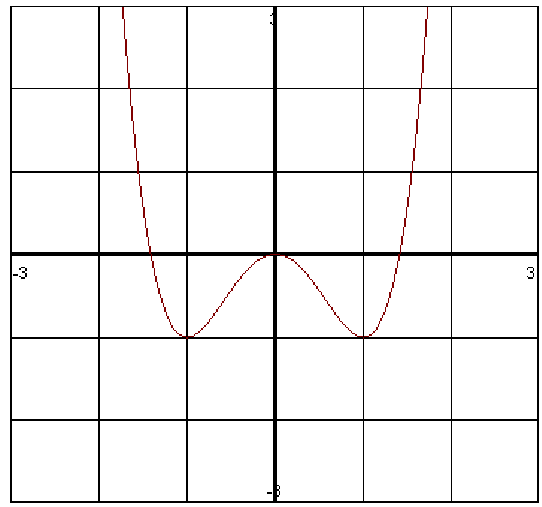
4.3 The First Derivative Test

Increasing: as x increases, y increases
Decreasing: as x increases, y decreases
Constant: as x increases, y does not change.

EXAMPLE: Use the graph of $f(x) = x^4 - 2x^2$ to determine the interval(s) of increasing, decreasing, or constant. Indicate all extrema (max and min)



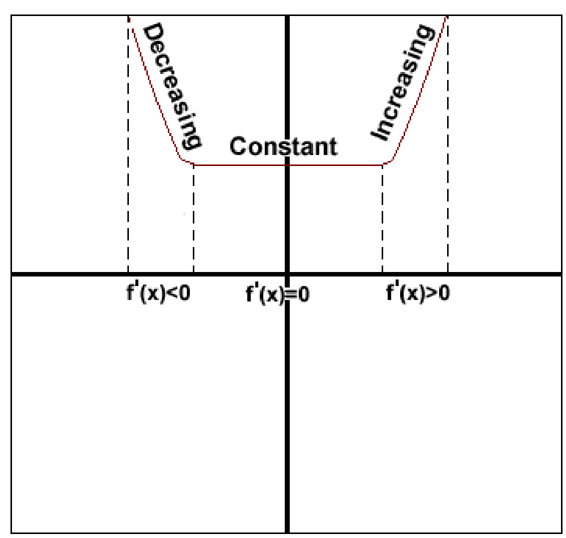
Here we were able to read our values off of the graph. Suppose a graph is not given. Do we need to always graph each function in order to find the interval(s) of increasing and decreasing? The answer is no. We can use what is called the First Derivative Test.

Corollary 3

If $f'(x) > 0$ for all x in (a, b) then f is increasing on [a, b].

If $f'(x) < 0$ for all x in (a, b) then f is decreasing on [a, b].

If $f'(x) = 0$ for all x in (a, b) then f is constant on [a, b].



How to find interval(s) of increasing, decreasing, constant with no graph

1.) Find the critical numbers in (a, b) to determine test intervals

2.) Determine the sign of $f'(x)$ at one test value in each of the intervals

3.) Use Corollary 3 to determine if f is increasing or decreasing.

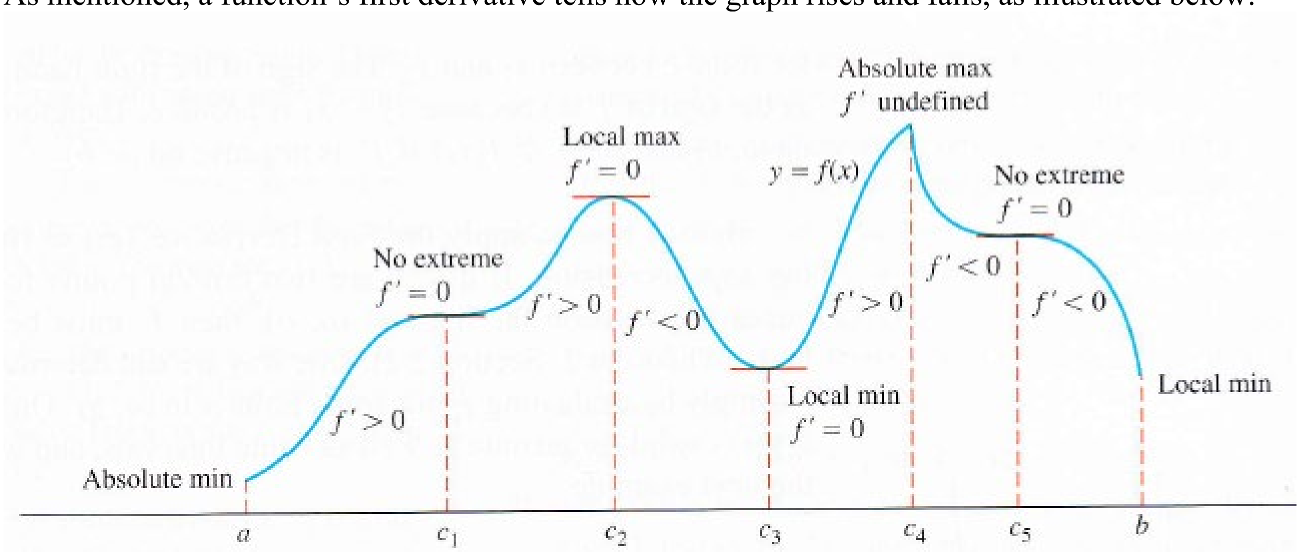
First Derivative Test for Local Extrema

If $f'(x)$ changes from – to + at c then f has a local minimum at x = c.

If $f'(x)$ changes from + to – at c then f has a local maximum at x = c.

If $f'(x)$ does not change on both sides of c then c is neither a min or a max.

As mentioned, a function's first derivative tells how the graph rises and falls, as illustrated below:



EXAMPLE: Find the critical points of: $f(x) = x^4 - 2x^2$. Identify the intervals on which *f* is increasing or decreasing. Find the function's local maximum and minimum.

EXAMPLE: Find the critical points of: $f(t) = 27t - t^3$. Identify the intervals on which f is increasing or decreasing. Find the function's local and absolute extreme values.

EXAMPLE: Find the critical points of: $f(x) = (x-2)e^{-3x}$. Identify the intervals on which *f* is increasing or decreasing. Find the function's local maximum and minimum.

EXAMPLE: Find the critical points of: $f(x)=\dfrac{\sqrt{9-x^2}}{x^2}$. Identify the intervals on which *f* is increasing or decreasing. Find the function's local and absolute extreme values.

EXAMPLE: Find all extrema and interval(s) of increasing and decreasing for $f(\theta) = \sin\theta\cos\theta$ on $[0,\ \pi]$.

EXAMPLE: Find the critical points of: $f(x) = 3x^{\frac{2}{3}}(x-4)$. Identify the intervals on which *f* is increasing or decreasing. Find the function's local and absolute extreme values.