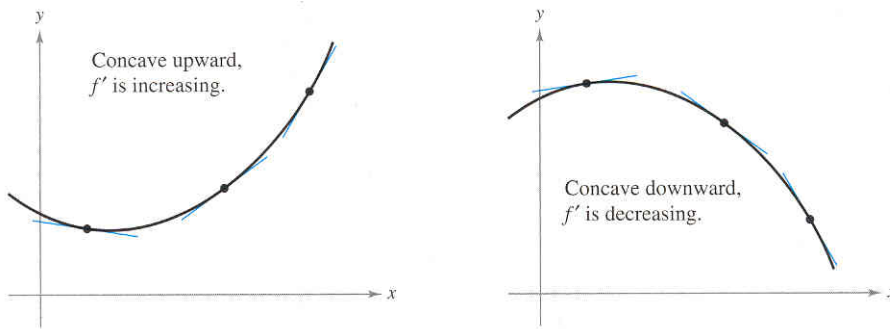


4.4 The Second Derivative Test and Curve Sketching

We can use the second derivative to tell us if a graph is concave up or concave down.



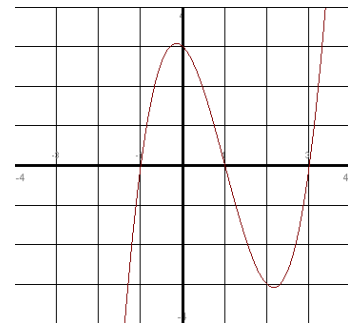
Second Derivative Test for Concavity

- 1.) If $f''(x) > 0$ for all x in I , then the graph of f is concave upward in I .
- 2.) If $f''(x) < 0$ for all x in I , then the graph of f is concave downward in I .

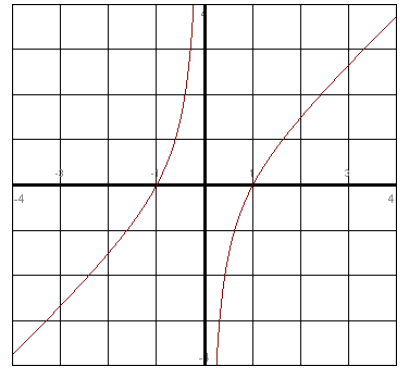
Hergert Number: Points where $f''(x) = 0$ or $f''(x)$ is undefined but are defined on $f(x)$.

Inflection point – the point at which the concavity changes. To find the inflection point, first find the Hergert numbers. Then test to see if there is a sign change, which indicates a change in concavity.

EXAMPLE: Use the graph to indicate the intervals of concavity and any points of inflection:



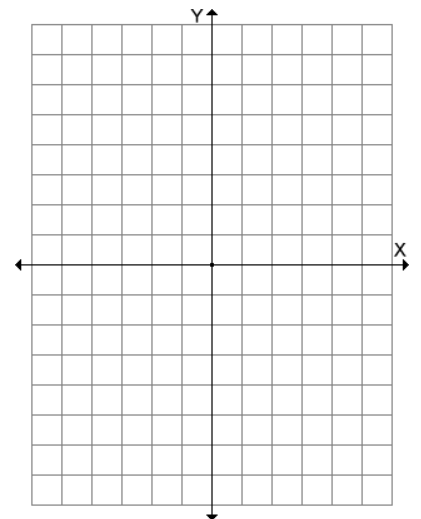
EXAMPLE: Use the graph to indicate the intervals of concavity and any points of inflection:



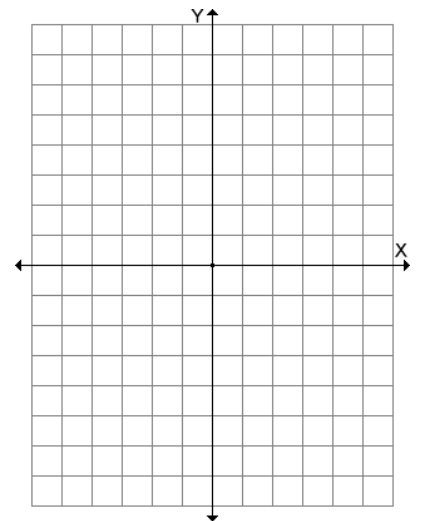
EXAMPLE: Given $f(x) = x^3(x - 4)$ find all points of inflection and interval(s) of concavity.

EXAMPLE: Given $f(x) = 2x^4 - 8x + 3$ find all points of inflection and interval(s) of concavity.

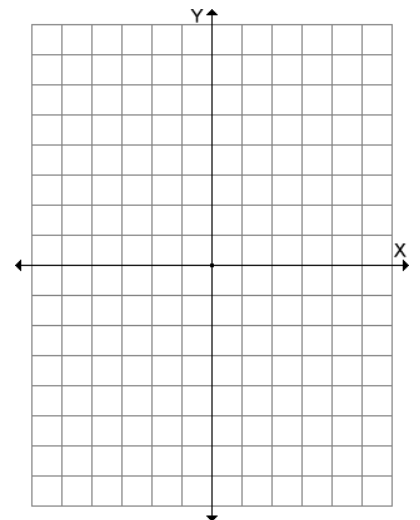
EXAMPLE: Find all extrema, interval(s) of increasing/decreasing, critical points, interval(s) of concavity, points of inflection, intercepts, asymptotes and use this information to graph $y = -\frac{1}{3}x^3 + x - \frac{2}{3}$.



EXAMPLE: Find all extrema, interval(s) of increasing/decreasing, critical points, interval(s) of concavity, points of inflection, intercepts, asymptotes and use this information to graph $y = x(x - 2)^3$.

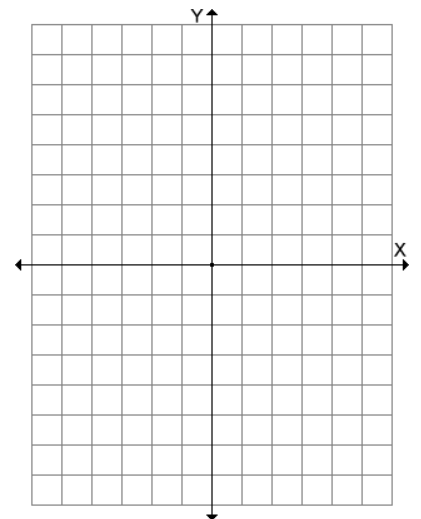


EXAMPLE: Find all extrema, interval(s) of increasing/decreasing, critical points, interval(s) of concavity, points of inflection, intercepts, asymptotes and use this information to graph $y = x^{\frac{2}{3}}(x^2 - 4)$.

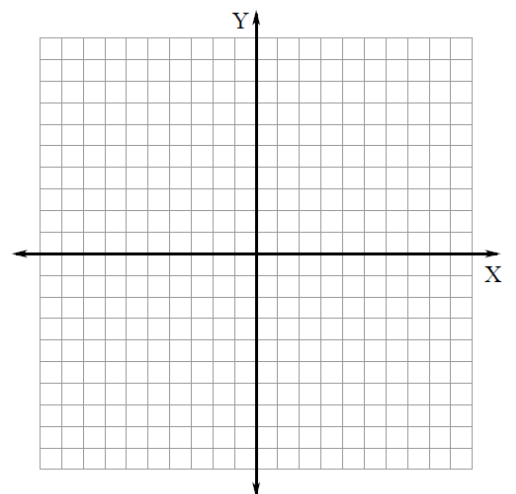


EXAMPLE: Given $y = x + \cos x$ find all points of inflection, critical points, interval(s) of concavity, interval(s) of increasing and interval(s) of decreasing on $[0, 2\pi]$. Then draw a sketch of this function.

EXAMPLE: Find all extrema, interval(s) of increasing/decreasing, critical points, interval(s) of concavity, points of inflection, intercepts, asymptotes and use this information to graph $y = \ln(5 - x^2)$.



EXAMPLE: Find all extrema, interval(s) of increasing/decreasing, critical points, interval(s) of concavity, points of inflection, intercepts, asymptotes and use this information to graph $y = \frac{-64}{x^2 - 64}$.



EXAMPLE: Sketch the curve $f(x)$ that meets the following conditions:

$$f(-2) = f(2) = 0 \text{ and } f(0) = 4$$

$$f'(-2) = f'(0) = f'(2) = 0$$

$$f''(-1) = f''(0) = f''(1) = 0$$

Sign changes for $f'(x)$

-	+	-	+
-2	0	2	

Sign changes for $f''(x)$

+	-	+
-1	1	

