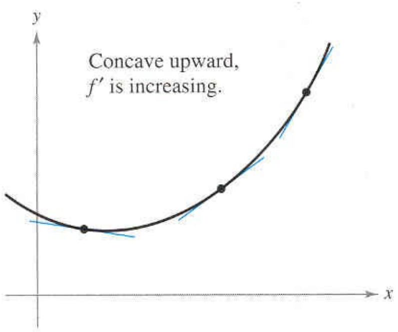
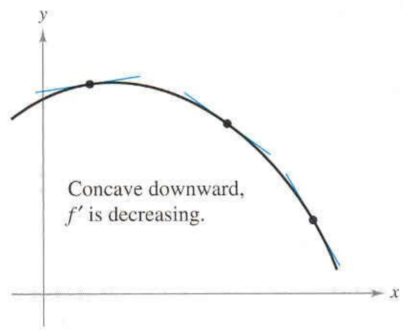
4.4 The Second Derivative Test and Curve Sketching

We can use the second derivative to tell us if a graph is concave up or concave down.





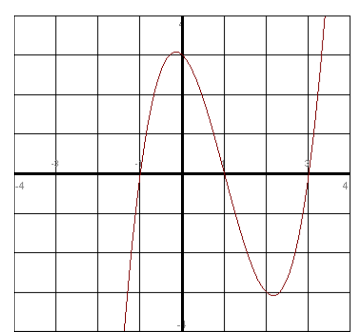
Second Derivative Test for Concavity

1.) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward in I.
2.) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward in I.

Hergert Number: Points where $f''(x) = 0$ or $f''(x)$ is undefined but are defined on $f(x)$.

Inflection point – the point at which the concavity changes. To find the inflection point, first find the Hergert numbers. Then test to see if there is a sign change, which indicates a change in concavity.

EXAMPLE: Use the graph to indicate the intervals of concavity and any points of inflection:



Concave up:______________

Concave down:____________

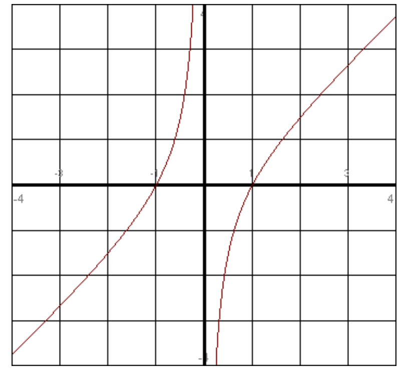
Inflection pt(s):____________

EXAMPLE: Use the graph to indicate the intervals of concavity and any points of inflection:

Concave up:______________

Concave down:____________

Inflection pt(s):____________



EXAMPLE: Given $f(x) = x^3(x-4)$ find all points of inflection and interval(s) of concavity.

Concave up:___________________

Concave down:_________________

Inflection pt(s):_________________

EXAMPLE: Given $f(x) = 2x^4 - 8x + 3$ find all points of inflection and interval(s) of concavity.

Concave up:___________________

Concave down:_________________

Inflection pt(s):_________________

EXAMPLE: Find all extrema, interval(s) of increasing/decreasing, critical points, interval(s) of concavity, points of inflection, intercepts, asymptotes and use this information to graph $y = -\frac{1}{3}x^3 + x - \frac{2}{3}$.

Critical numbers:________________

Increasing:______________________

Decreasing:_____________________

Relative Max:___________________

Relative Min:___________________

Hergert Numbers:________________

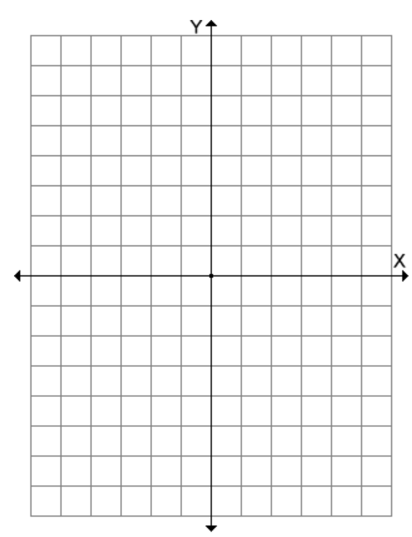
Concave up:____________________

Concave down:__________________

Inflection pt(s):_________________

x-intercepts:_____________________

y-intercepts:_____________________



EXAMPLE: Find all extrema, interval(s) of increasing/decreasing, critical points, interval(s) of concavity, points of inflection, intercepts, asymptotes and use this information to graph $y = x(x-2)^3$.

Critical numbers:_________________

Increasing:______________________

Decreasing:_____________________

Relative Max:___________________

Relative Min:___________________

Hergert Numbers:________________

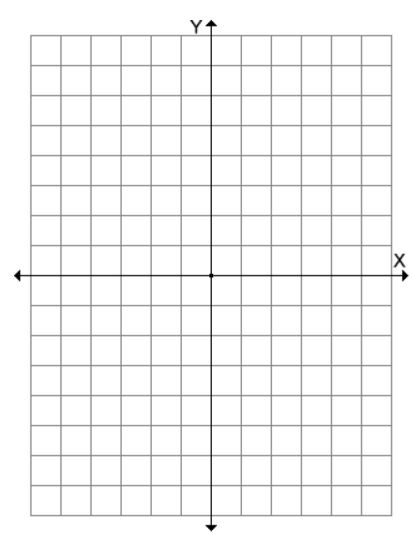
Concave up:____________________

Concave down:__________________

Inflection pt(s):__________________

x-intercepts:_____________________

y-intercepts:_____________________



EXAMPLE: Find all extrema, interval(s) of increasing/decreasing, critical points, interval(s) of concavity, points of inflection, intercepts, asymptotes and use this information to graph $y = x^{\frac{2}{3}}\left(x^2 - 4\right)$.

Critical numbers:________________

Increasing:______________________

Decreasing:_____________________

Relative Max:__________________

Relative Min:__________________

Hergert Numbers:_______________

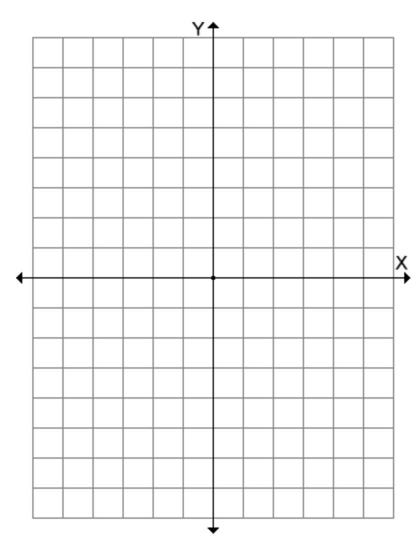
Concave up:____________________

Concave down:_________________

Inflection pt(s):_________________

x-intercepts:____________________

y-intercepts:____________________



EXAMPLE: Given $y = x + \cos x$ find all points of inflection, critical points, interval(s) of concavity, interval(s) of increasing and interval(s) of decreasing on $[0,\ 2\pi]$. Then draw a sketch of this function.

Critical numbers:________________

Increasing:______________________

Decreasing:_____________________

Relative Max:__________________

Relative Min:__________________

Hergert Numbers:_______________

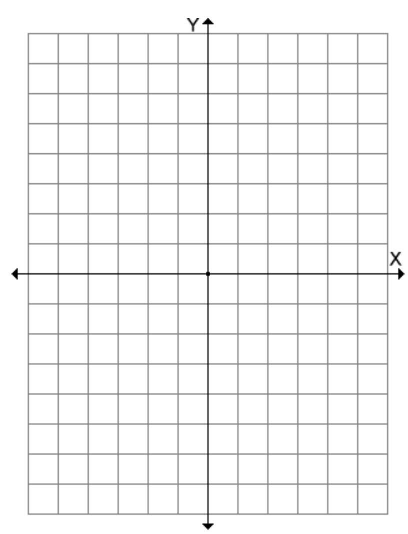
Concave up:____________________

Concave down:_________________

Inflection pt(s):_________________

x-intercepts:_____________________

y-intercepts:_____________________



EXAMPLE: Find all extrema, interval(s) of increasing/decreasing, critical points, interval(s) of concavity, points of inflection, intercepts, asymptotes and use this information to graph $y = \ln\left(5 - x^2\right)$.

Critical numbers:________________

Increasing:______________________

Decreasing:_____________________

Relative Max:___________________

Relative Min:___________________

Hergert Numbers:________________

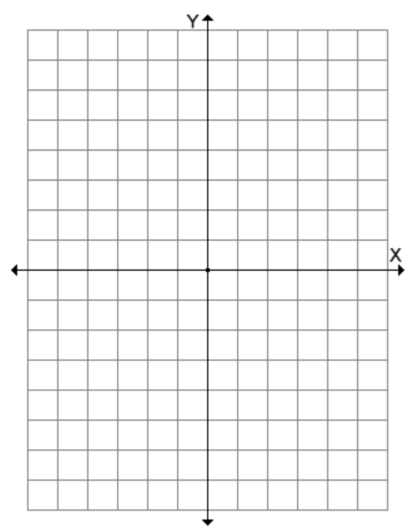
Concave up:____________________

Concave down:__________________

Inflection pt(s):__________________

x-intercepts:_____________________

y-intercepts:_____________________



EXAMPLE: Sketch the curve $f(x)$ that meets the following conditions:

$f(-2) = f(2) = 0$ and $f(0) = 4$
$f'(-2) = f'(0) = f'(2) = 0$
$f''(-1) = f''(0) = f''(1) = 0$

Sign changes for $f'(x)$

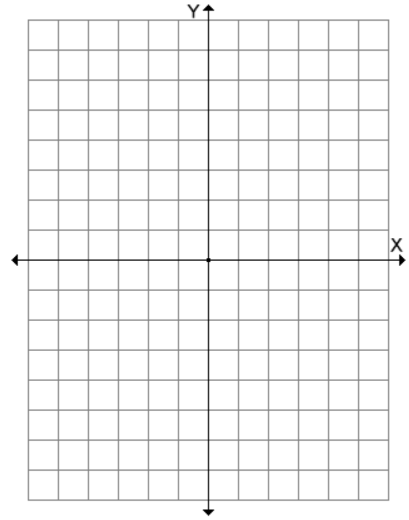
–	+	–	+

$-2 \qquad 0 \qquad 2$

Sign changes for $f''(x)$

+	–	+

$-1 \qquad 1$



EXAMPLE: Sketch the curve $f(x)$ that meets the following conditions:

$f(2) = 0$ and $f(0) = -3$
$f'(-1) = f'(1) = 0$
$f''(0) = 0$

Sign changes for $f'(x)$

+	–	+

$-1 \qquad 1$

Sign changes for $f''(x)$

–	+

0