4.5 L'Hopital's Rule

In this section we will be looking at limits again. Particularly we are looking at limits that result in indeterminate forms (expressions that cannot be calculated). In this case we can apply L'Hopital's Rule in order to evaluate limits that result in indeterminate forms.

Types of indeterminate forms: $\frac{0}{0}, \frac{\infty}{\infty}, \infty \cdot 0, \infty - \infty$

L'Hopital's Rule: Suppose that $f(a) = g(a) = 0$ and that f and g are differentiable on an open interval I containing a, and that $g'(x) \neq 0$ on I if $x \neq a$. Then:

$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$, assuming that the limit on the right side of this equation exists.

Using L'Hopital's Rule:

To find $\lim_{x \to a} \frac{f(x)}{g(x)}$ by L'Hopital's Rule, keep taking the derivative of f and g as long as you keep ending up with the form 0/0 at $x = a$. As soon as one of these derivatives is not zero at $x = a$ then stop taking derivatives and find the limit. Remember the L'Hopital's rule does not apply when either the numerator or denominator has a finite nonzero limit.

EXAMPLE: Use L'Hopital's Rule to evaluate $\lim_{x \to 3} \frac{x-3}{x^2-9}$. Then evaluate it using limit rules that we studied in Chapter 2.

EXAMPLE: Use L'Hopital's Rule to evaluate $\lim_{x\to 0}\frac{1-\cos x}{4x^2}$. Then evaluate it using limit rules that we studied in Chapter 2.

EXAMPLE: Use L'Hopital's Rule to evaluate $\lim_{x\to 1}\frac{x^3-1}{4x^3-7x+3}$.

EXAMPLE: Use L'Hopital's Rule to evaluate $\lim_{x\to\infty}\frac{x-3x^2}{4x^2+9x}$.

EXAMPLE: Use L'Hopital's Rule to evaluate $\lim_{x\to 0}\frac{\sin\left(3x^2\right)}{5x}$.

EXAMPLE: Use L'Hopital's Rule to evaluate $\lim_{x\to 0}\dfrac{x(1-\cos x)}{3x-\sin 3x}$.

EXAMPLE: Use L'Hopital's Rule to evaluate $\lim_{x\to 0}\frac{x\cdot 5^x}{3^x-1}$.

EXAMPLE: Use L'Hopital's Rule to evaluate $\lim_{x\to 0}\frac{\sqrt{4x+16}-4}{x}$.

EXAMPLE: Use L'Hopital's Rule to evaluate $\lim_{x\to \infty}\frac{e^{2x}-x^3}{e^{2x}+1}$.