5.1 Antiderivatives

Suppose we had $f(x) = x^3$ and we wanted to find the derivative. We can use the power rule: $f'(x) = 3x^2$. What if we started with the derivative and we wanted to get back to the original function. This will involve the antiderivative.

Antiderivative: A function f is an antiderivative of f on an interval I if $f'(x) = f(x)$ for all x in I. The notation that is used for the antiderivative is the following: $\int$

Antiderivative formulas and properties:

$\int x^n \; dx = \frac{x^{n+1}}{n+1} + C$ Where $n \neq -1$, $\int 0 \; dx = C$, $\int k \; dx = kx + C$

$\int k \cdot f(x) \; dx = k\int f(x) \; dx$, $\int f(x) + g(x) \; dx = \int f(x) \; dx + \int g(x) \; dx$

$\int \frac{1}{x} \; dx = \ln|x| + C$, $x \neq 0$, $\int e^{kx} dx = \frac{1}{k} e^{kx} + C$

$\int \cos kx \; dx = \frac{1}{k} \sin kx + C$, $\int \sin kx \; dx = -\frac{1}{k} \cos kx + C$

$\int \sec^2 kx \; dx = \frac{1}{k} \tan kx + C$, $\int \sec kx \cdot \tan kx \; dx = \frac{1}{k} \sec kx + C$

$\int \csc^2 kx \; dx = -\frac{1}{k} \cot kx + C$, $\int \csc kx \cdot \cot kx \; dx = -\frac{1}{k} \csc kx + C$

$\int \frac{1}{\sqrt{1-k^2x^2}} \; dx = \frac{1}{k} \sin^{-1} kx + C$, $\int \frac{1}{1+k^2x^2} \; dx = \frac{1}{k} \tan^{-1} kx + C$, $\int \frac{1}{x\sqrt{k^2x^2 - 1}} \; dx = \sec^{-1} kx + C$

$\int a^{kx} dx = \left(\frac{1}{k \ln a} \right) a^{kx} + C$, $a > 0$, $a \neq 1$

EXAMPLE: Find the antiderivative for the function $4x^3$ when C equals 0.

EXAMPLE: Find the antiderivative for the function x^{-8} when C equals 0.

EXAMPLE: Find the antiderivative for the function $\sqrt[10]{x}+\frac{1}{\sqrt[10]{x}}$ when C equals 0.

EXAMPLE: Find the antiderivative for the function $\sin(12x)+\cos(12x)$ when C equals 0.

EXAMPLE: Find the antiderivative for the function $-\csc^2\left(\frac{2x}{5}\right)$ when C equals 0.

EXAMPLE: Find the antiderivative for the function $\frac{1}{x\sqrt{169x^2-1}}$ when C equals 0.

EXAMPLE: Find the antiderivative for the function $e^{3x}-e^{\frac{x}{7}}-\frac{1}{x}$ when C equals 0.

EXAMPLE: Find the indefinite integral: $\int (4x^3 + 6x^2 - 3)\ dx$ and check your answer.

EXAMPLE: Find the indefinite integral: $\int \frac{3}{x^2} - \frac{1}{5\sqrt{x}} + \frac{3}{4} dx$.

EXAMPLE: Find the indefinite integral: $\int x^{-3}(x+1) + \sin(3x)\ dx$

EXAMPLE: Find the indefinite integral: $\int \frac{5}{e^{2x}} + 3^{4x} \; dx$.

EXAMPLE: Find the indefinite integral: $\int \left(\frac{x\sqrt{x} + 2 \cdot \sqrt[3]{x} - x}{x^2} \right) dx$.

EXAMPLE: Find the indefinite integral: $\int (\theta^2 + \sec^2 7\theta) \; d\theta$.

EXAMPLE: Find the indefinite integral: $\int \left(\frac{\cos(4\theta)}{1-\cos^2(4\theta)} \right) d\theta$.

EXAMPLE: Find the indefinite integral: $\int \frac{dx}{1+4x^2}$.

EXAMPLE: Solve the initial value problem: Given: $\frac{dy}{dx} = 6x^2$ and $y(0) = -1$.

EXAMPLE: Solve the initial value problem: Given $\frac{d^2 y}{dx^2} = x^2$ and $y'(0) = 6$ and $y(0) = 3$.

EXAMPLE: Solve the initial value problem: Given $\frac{d^2 r}{d\theta^2} = \sin 5\theta$ and $r'(0) = 1$ and $r(0) = 6$.