5.3 The Definite Integral

Let $\|P\|$ be the norm, which represents the largest subinterval (partition), or the largest width.

$\|P\| = \Delta x = \dfrac{b-a}{n}$. As $\|P\| \to 0$, $n \to \infty$.

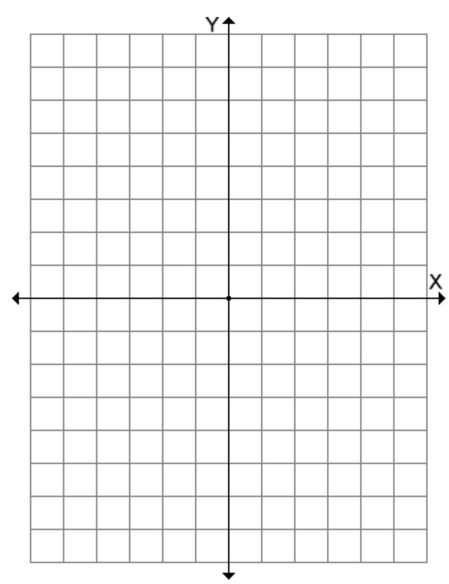
$$\lim_{\|P\|\to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)dx.$$

EXAMPLE: Evaluate using the limit process: $\int_{-2}^{3} x\ dx$.

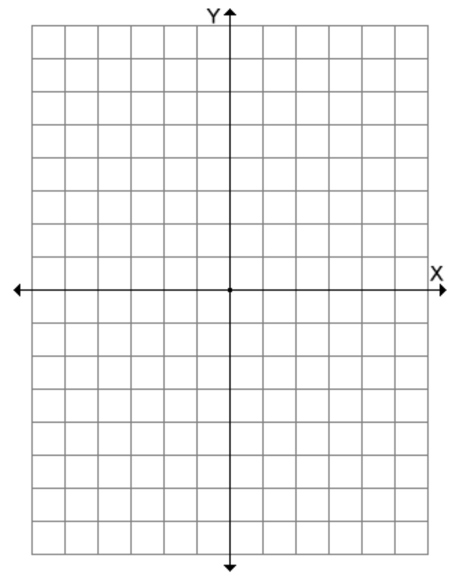
EXAMPLE: Change the following into a definite integral: $\lim_{\|P\|\to 0} \sum_{i=1}^{n} 6c_i(4-c_i)^2 \Delta x_i$ where P is a partition of [0, 4].

EXAMPLE: Change the following into a definite integral: $\lim_{\|P\|\to 0} \sum_{i=1}^{n} \left(\frac{3}{c_i^{\,2}}\right) \Delta x_i$ where P is a partition of [1, 3].

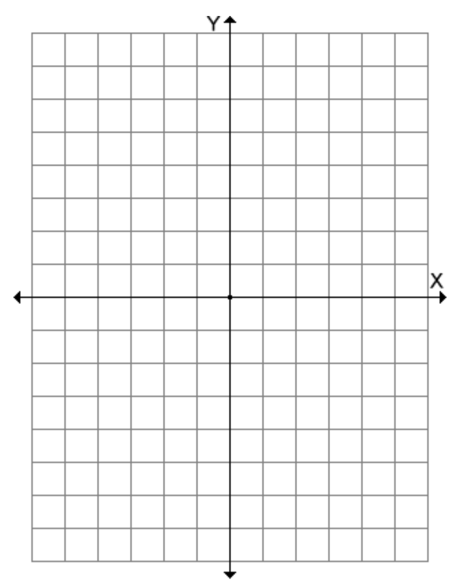
EXAMPLE: Graph the integrand $\int_0^2 -2x+4 \; dx$, and use areas to evaluate the integral.



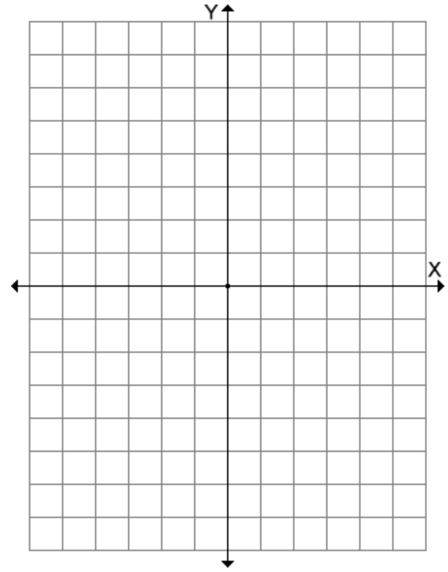
EXAMPLE: Graph the integrand $\int_{-3}^{3} \sqrt{9-x^2}\ dx$, and use areas to evaluate the integral.



EXAMPLE: Graph the integrand $\int_{0}^{2} x+1\ dx$, and use areas to evaluate the integral.



EXAMPLE: Graph the integrand $\int_{-1}^{3} 2-|x-1| \ dx$, and use areas to evaluate the integral.



Properties of Definite Integrals (Assume f and g are integrable on [a, b]).

1.) $\int_{a}^{a} f(x)dx = 0$ From a to a we have a rectangle with a width of 0, so the area is 0.

2.) $\int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx$ If we switch the order of a and b the area changes sign.

3.) $\int_{a}^{b} f(x)dx = \int_{a}^{c} f(x)dx + \int_{c}^{b} f(x)dx$ We can split up the area in to two separate areas.

4.) $\int_{a}^{b} k \cdot f(x)dx = k\int_{a}^{b} f(x)dx$ We are allowed to take a constant k out of the integral.

5.) $\int_{a}^{b} [f(x) \pm g(x)]dx = \int_{a}^{b} f(x)dx \pm \int_{a}^{b} g(x)dx$ We can do two separate integrals.

EXAMPLE: Given $\int_2^4 x^3\,dx = 60$, $\int_2^4 x\,dx = 6$, and $\int_2^4 dx = 2$ evaluate the following using properties of definite integrals:

a.) $\int_2^2 x^3\,dx$

b.) $\int_2^4 15\,dx$

c.) $\int_2^4 \left(x^3 + 4\right) dx$

d.) $\int_4^2 x(2 - x^2)\,dx$

EXAMPLE: Given $\int_1^3 f(x)\ dx = -2$, $\int_1^6 f(x)\ dx = 5$, and $\int_1^6 g(x)\ dx = 7$, evaluate the following using properties of definite integrals:

a.) $\int_3^3 g(x)\ dx$

b.) $\int_6^1 3f(x)\ dx$

c.) $\int_3^6 f(x)\ dx$

d.) $\int_6^1 \left[f(x) - g(x)\right]\ dx$

e.) $\int_1^6 \left[-2f(x) + g(x)\right]\ dx$