1.1 Review of Functions

Before we jump in calculus, we will first do a quick review of functions, graphs, and equations

Function Definition: For each input (x) there can only be one output (y).

Function notation: $f(x)$ which means "f of x". This does not mean f times x. It means that we have a function called f which contains the variable x.

EXAMPLE: Given the function $f(x) = 2x - 5$, find the following:

a.) Find $f(3)$. Solve $f(x) = 7$.

b.) $f(x+3)$

c.) $f(x) + f(3)$

d.) $f(x+h)$

Domain: (input) all the x-values that make the equation defined
Defined: There is no division by zero or square roots of negative numbers
Range: (output) all y-values that a graph uses.

EXAMPLE: Find the domain: $y = 2x - 5$

EXAMPLE: Find the domain: $y = \dfrac{1}{2x-5}$

EXAMPLE: Find the domain: $y = \sqrt{2x-5}$

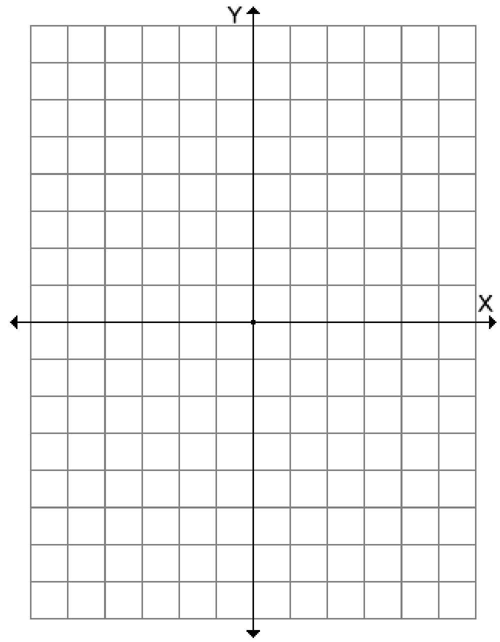
EXAMPLE: Find the domain: $y = \dfrac{1}{\sqrt{2x-5}}$

EXAMPLE: Find the domain: $y = \dfrac{1}{x^2+9}$

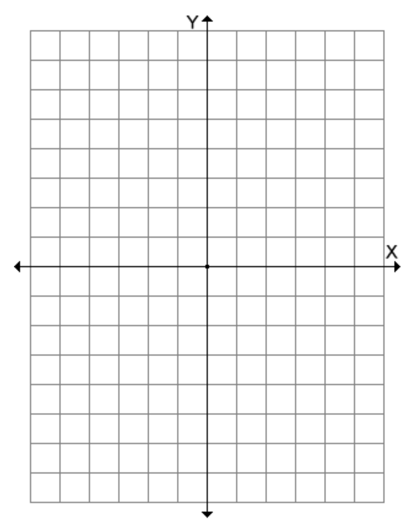
EXAMPLE: Graph the equation $4x^2 + y = 4$. Let x = -2, -1, 0, 1, 2. Then indicate the intercepts.

It doesn't matter if you've never graphed a quadratic equation before. We plot this one the same way we plot lines. Once again, we need to first solve for y: $y = -4x^2 + 4$. Now we need to set up a table like this:

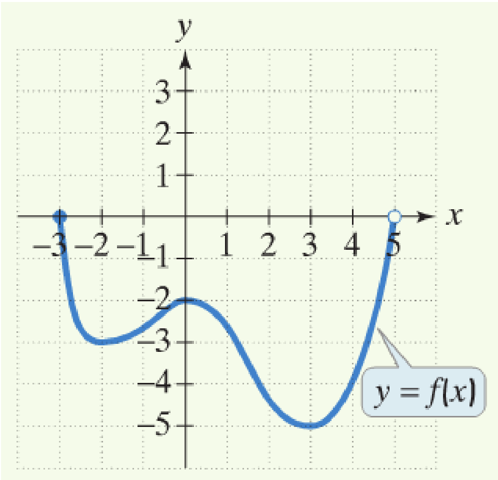
x	$y = -4x^2 + 4$	(x, y)
-2		
-1		
0		
1		
2		



EXAMPLE: Find the x and y intercepts and use them to graph the following equation: 6x – 3y + 15 = 0.



EXAMPLE: Use the graph below to answer the following questions



a.) Indicate the interval(s) of which f is increasing

b.) Indicate the interval(s) of which f is decreasing

c.) List the number(s) where f has a relative minimum.

d.) What is the relative maximum(s)?

e.) What is the relative minimum(s)?

f.) What is the domain?

g.) What is the range?

Composite Functions – a way of combining two functions

$(f \circ g)(x) = f(g(x))$ This is pronounced "f of g of x" DOES NOT MEAN F TIMES G!!!
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These are not multiplications. The $(f \circ g)(x)$ means we place the g function into the f function.
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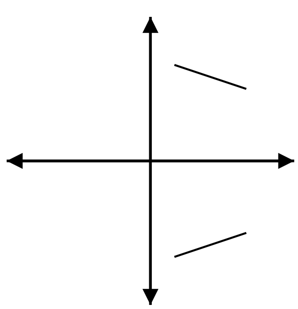
EXAMPLE: Given: $f(x) = x + 3$ and $g(x) = 2x^2 - 1$ find the following:

$(f \circ g)(x)$

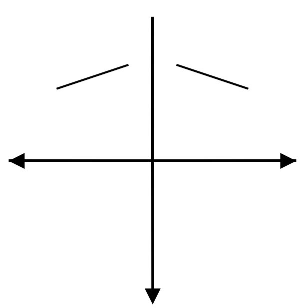
$(g \circ f)(x)$

Symmetry

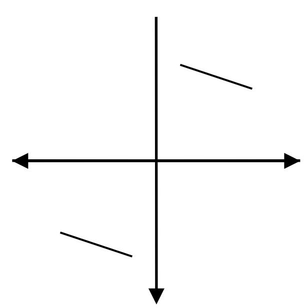
Think if symmetry as a fold line. If a graph can be folded on top of itself and everything overlaps, then it has symmetry. The fold line that allows this to happen is called the line of symmetry. Below is the three types of symmetry that is possible.



x – axis symmetry



y-axis symmetry



origin symmetry

Looking at the above drawings we can come up with relationships. Lets begin with a point (x, y).

Two points symmetric about the x-axis would be (x, y) and (x, -y).
Two points symmetric about the y-axis would be (x, y) and (-x, y).
Two points symmetric about the origin would be (x, y) and (-x, -y).

How to test for symmetry without graphing

x-axis: Replace y with –y in the original equation. If it simplifies to the original equation, it has x-axis symmetry.

y-axis: Replace x with –x in the original equation. If it simplifies to the original equation, it has y-axis symmetry.
.
origin: Replace x with –x and y by -y in the original equation. If it simplifies to the original equation, it has origin symmetry.

EXAMPLE: Test the following equation for symmetry: $y = x^3 - 4x$ and find the intercepts.

x-int: replace y with zero and solve for x.

y-int: replace x with zero and solve for y.

x-axis: Replace y with –y in the original equation.

y-axis: Replace x with –x in the original equation.

Origin: Replace x with –x and y by -y in the original equation.

Even and Odd functions

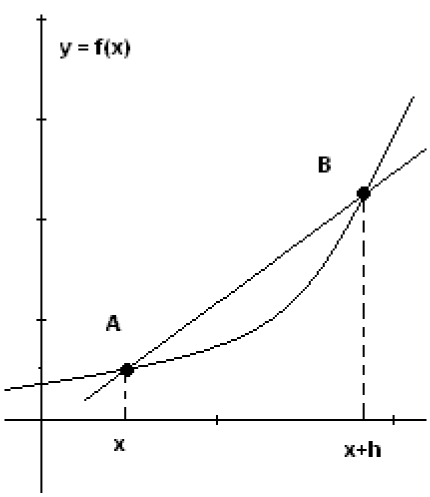
If $f(-x) = f(x)$ then the function is even, and symmetric to the y-axis.
If $f(-x) = -f(x)$ then the function is odd, and symmetric to the origin.

EXAMPLE: Determine whether the following are even, odd, or neither.

a.) $f(x) = x^4 + 7$

b.) $f(x) = 6x^5 - x^3$

Difference Quotient



If we wanted to find the slope of a curved line, the only way we can do this is by estimating it with a straight line. We will start with one point and then move over by a small amount h. Now we will use the slope formula. In the picture we have two points, A and B. The coordinates for these are: $(x, f(x))$ and $(x+h, f(x+h))$.

The slope, also called the **difference quotient** is: $\dfrac{f(x+h)-f(x)}{h}$

In calculus we will try to minimize h so that it is so small that we end up at a point, which will be the exact slope of the curved line at x.

EXAMPLE: Let $f(x) = 3x^2 - x + 1$. Find the difference quotient.