Section 1.3 Trigonometric Functions

Angles are measured a couple of different ways. The first unit of measurement is a **degree** in which 360° (degrees) is equal to one revolution.

Another unit of measurement for angles is **radians**. In radians, 2π is equal to one revolution. So a conversion between radians and degrees is $2\pi = 360^\circ$, or $\pi = 180^\circ$.

When converting from degrees to radians:

Multiply your degrees by $\frac{\pi}{180}$

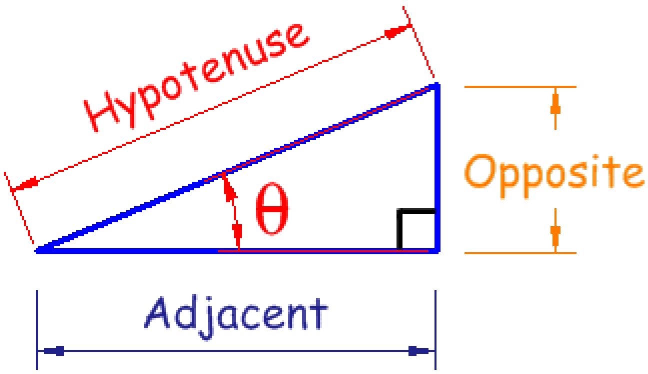
When converting from radians to degrees:

Multiply your radians by $\frac{180}{\pi}$

EXAMPLE: Convert 60° to radians.

EXAMPLE: Convert $\frac{4\pi}{3}$ into degrees.

Right Triangle Trig Definitions



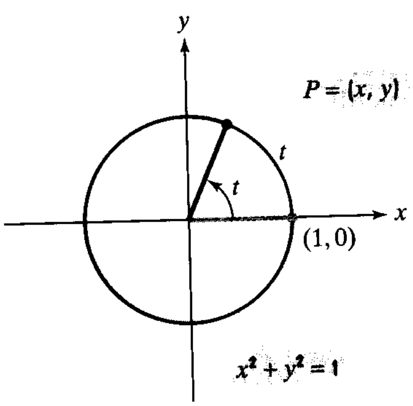
$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \csc\theta = \frac{hypotenuse}{opposite}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} \qquad \sec\theta = \frac{hypotenuse}{adjacent}$$

$$\tan\theta = \frac{opposite}{adjacent} \qquad \cot\theta = \frac{adjacent}{opposite}$$

EXAMPLE: Given $\cos\theta = -\frac{1}{4}$ and $\sin\theta < 0$, find the exact value of the six trig functions.

A **unit circle** is a circle centered at the origin with a radius of 1. It is shown in the drawing below. Here the letter t represents an angle measure. The point P = (x, y) represents a point on the unit circle.



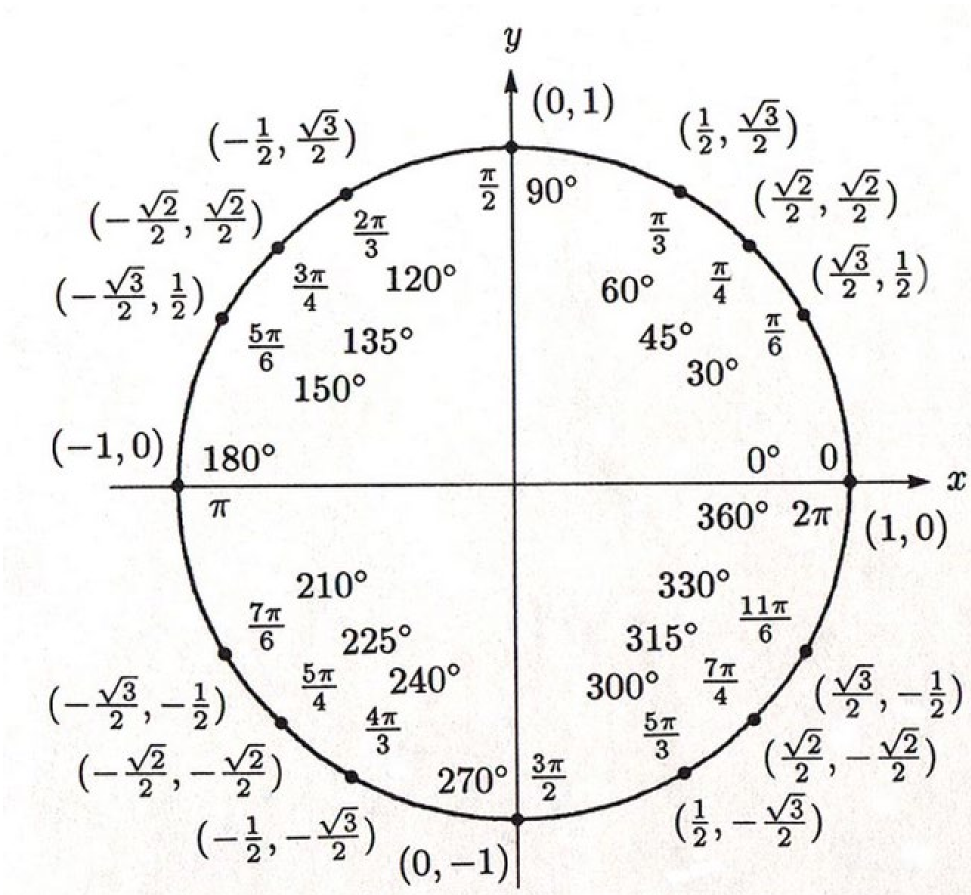
The following definitions are given based on this picture.

$\cos(t) = x$ $\quad\sec(t) = \frac{1}{x}$

$\sin(t) = y$ $\quad\csc(t) = \frac{1}{y}$

$\tan(t) = \frac{y}{x}$ $\quad\cot(t) = \frac{x}{y}$

The angle t can be any angle between 0 and 360 degrees. There are certain places on the unit circle in which we have exact values. The values are given below. This unit circle contains both degrees and radians.

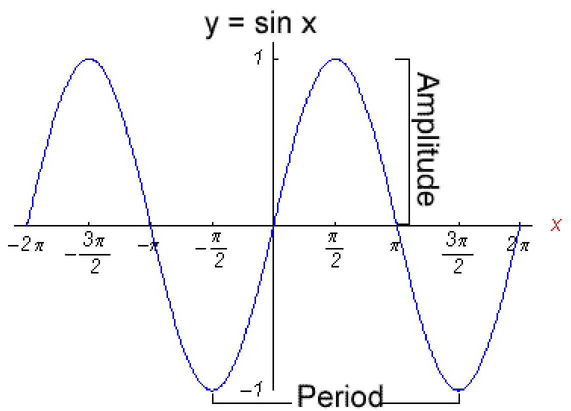


EXAMPLE: Find the exact value of $\cos 135^\circ$ without using the cosine function on a calculator.

EXAMPLE: Find the exact value of $\tan\left(-\frac{2\pi}{3}\right)$ without using the tangent function on a calculator.

EXAMPLE: Find the exact value of $\sin\left(-\frac{11\pi}{3}\right)$ without using the sine function on a calculator.

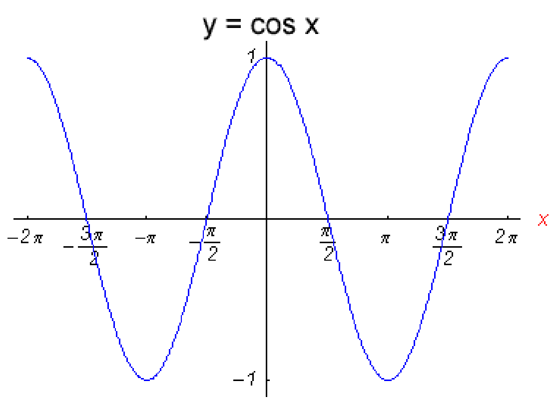
Graphs of Sine and Cosine



Period: How long it takes the graph to repeat itself
For sine graphs, the period is 2π .

$$\textbf{Amplitude} = \frac{Highest\ \ value - Lowest\ \ value}{2}$$

For the regular sine graph the amplitude is 1.



The period for cosine graphs is 2π

The amplitude for a regular cosine graph is 1.

General Form of a Sine or Cosine Equation:

$y = A\sin(Bx - C)$ or $y = A\cos(Bx - C)$

Amplitude = $|A|$, **Period** = $\frac{2\pi}{B}$, **Phase Shift** = $\frac{opp\ sign\ of\ C}{B}$

The **phase shift** is a shift of the graph to the left or to the right.

EXAMPLE: Identify the amplitude, period, phase shift and graph of $y = 3\cos\left(3x - \frac{\pi}{2}\right)$. (Graph 1 period).

Period:

Amplitude:

Phase Shift:

Quarter Point:

List of Trigonometric Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ $\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $\sin^2\theta = 1 - \cos^2\theta$ $\cos^2\theta = 1 - \sin^2\theta$

$\sec^2\theta = 1 + \tan^2\theta$ $\tan^2\theta = \sec^2\theta - 1$ $\csc^2\theta = 1 + \cot^2\theta$ $\cot^2\theta = \csc^2\theta - 1$

$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\cos 2\theta = 1 - 2\sin^2\theta$ $\cos 2\theta = 2\cos^2\theta - 1$

Proving a Trigonometric Identity

Although we will not specifically be doing these types of problems in Calculus, these problems are good review of your trig identities.

EXAMPLE: Establish the identity: $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = 2\sec x$.

Solving Trigonometric Equations

EXAMPLE: Solve the equation: $2\cos^2 x + \cos x - 1 = 0$ on $[0,\ 2\pi)$.

EXAMPLE: Solve the equation: $\sin x \cos^2 x = 2\sin x$ on $[0,\ 2\pi)$.

EXAMPLE: Solve the equation: $2\sin^2\theta - \cos 2\theta = 0$ on $[0,\ 360^\circ)$.