

# 1.4 Inverse Functions

## One-To-One Function

A function  $f$  is a one-to-one function if for  $a$  and  $b$  in the domain of  $f$ , if  $a \neq b$ , then  $f(a) \neq f(b)$ , or equivalently, if  $f(a) = f(b)$  then  $a = b$ . In other words, for each  $y$  value there can only be one  $x$  value.

EXAMPLE: For each function below, determine whether each is one-to-one.

$\{(1, 2), (3, 7), (2, 9), (8, 11)\}$

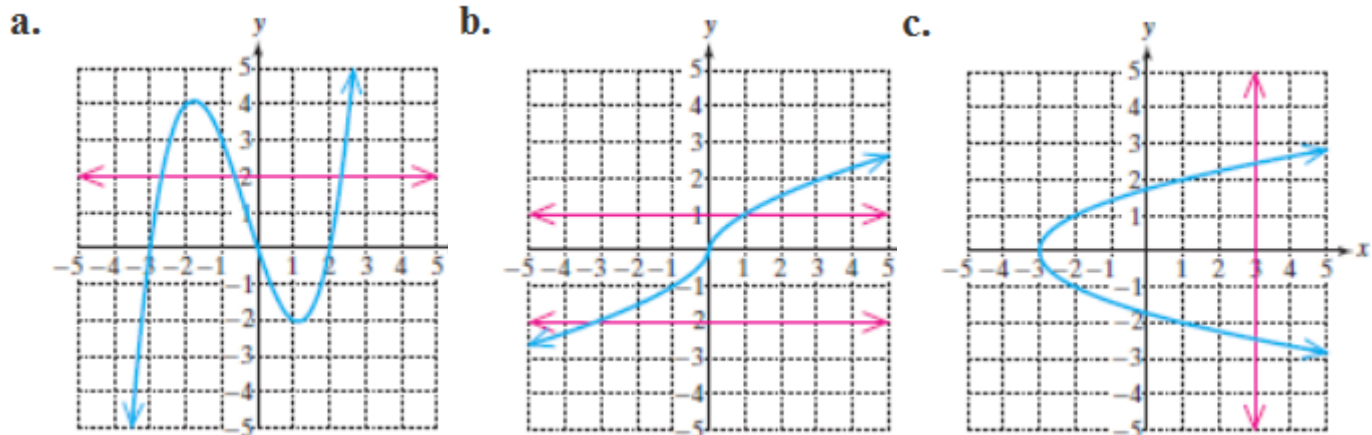
$\{(-3, 4), (5, 6), (7, 4), (-2, 3)\}$

$\{(-2, 4), (-1, 6), (0, 3), (-2, 8)\}$

## Horizontal Line Test

If you pass a horizontal line and it hits the graph at only one place, then it is one-to-one.

EXAMPLE: Use the horizontal line test to determine if the graph below defines  $y$  as a one-to-one function.



**Inverse Function:**

Notation to write “the inverse of  $f(x)$ ” is  $f^{-1}(x)$ . If two functions  $f$  and  $f^{-1}$  are inverses then the following must be true:  $(f \circ f^{-1})(x) = x$  and  $(f^{-1} \circ f)(x) = x$ . Note that these functions must be one-to-one.

EXAMPLE: Given  $f(x) = 2x - 1$  and  $f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$  verify that they are inverses.

**How to find an inverse algebraically:**

Step 1: Replace  $f(x)$  with  $y$ .

Step 2: Switch  $x$  and  $y$ .

Step 3: Solve for  $y$ .

Step 4: Replace  $y$  with  $f^{-1}(x)$ .

EXAMPLE: Given  $f(x) = \sqrt{x+7}$  find  $f^{-1}(x)$ .

Step 1:

Step 2:

Step 3:

Step 4:

### Inverse Trigonometric Functions

From trigonometry we know that  $\sin 30^\circ = \frac{1}{2}$ . We put in an angle and get a value as a result. In inverse trig functions we put in the value and get an angle:  $\sin^{-1} \frac{1}{2} = 30^\circ$ . So here we put in the value of one half and got 30 degrees as a result. We are not allowed to put any number into our inverse trig functions. There are restrictions on the domain that are given in the following table:

	Domain	Range
$y = \sin^{-1} x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

EXAMPLE: Find  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ .

EXAMPLE: Find  $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ .

EXAMPLE: Find  $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ .

EXAMPLE: Find  $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$ .

EXAMPLE: Find the exact value:  $\cos^{-1}\left(\cos\frac{\pi}{3}\right)$ .