1.5 Exponential and Logarithmic Functions

The first part of this section will be a review over exponents. Listed below are rules for exponents.

Exponent Laws:

$a^s \cdot a^t = a^{s+t}$ Example: $2^5 \cdot 2^3 = 2^{5+3} = 2^8$

$\frac{a^s}{a^t} = a^{s-t}$ Example: $\frac{2^6}{2^3} = 2^{6-3} = 2^3$

$(a^s)^t = a^{s \cdot t}$ Example: $(2^3)^5 = 2^{3 \cdot 5} = 2^{15}$

$a^{-s} = \frac{1}{a^s}$ Example: $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

$\left(\frac{a}{b}\right)^{-s} = \left(\frac{b}{a}\right)^s$ Example: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

EXAMPLE: Use laws of exponents to simplify: $\left(17^{\sqrt{2}}\right)^{\frac{\sqrt{2}}{2}}$.

EXAMPLE: Use laws of exponents to simplify: $\left(4^7 \cdot 4^3\right)^{\frac{1}{5}}$.

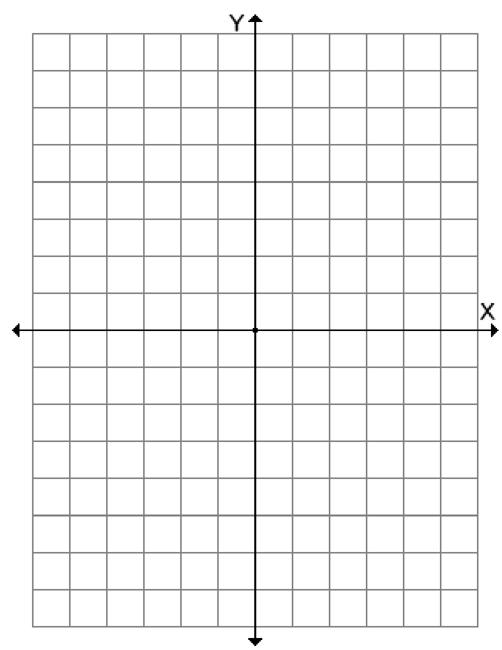
EXAMPLE: Simplify: $\left(\frac{3x^4y^{-2}}{x^{-3}y}\right)^{-4}\left(\frac{y}{x}\right)^{-2}$ and write with positive exponents.

Exponential function: $y = b^x$

We will look at a specific exponential function to see its characteristics. To do this we will make a table. Then we will plot the points. The graph will be a curved line:

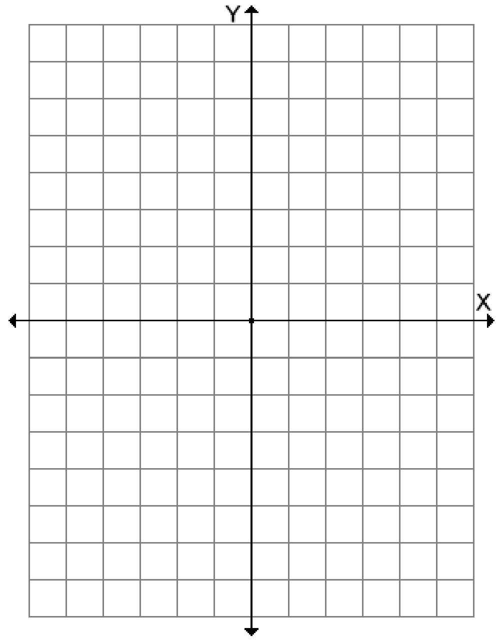
Graph of $y = 2^x$

x	$y = 2^x$	(x, y)
-2		
-1		
0		
1		
2		



EXAMPLE: Graph using transformations: $y = -2^x$.
Indicate the domain and range. State the horizontal asymptote.

Domain:

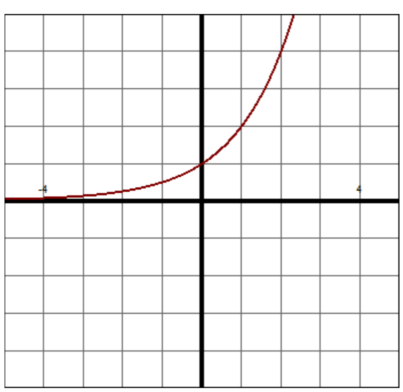


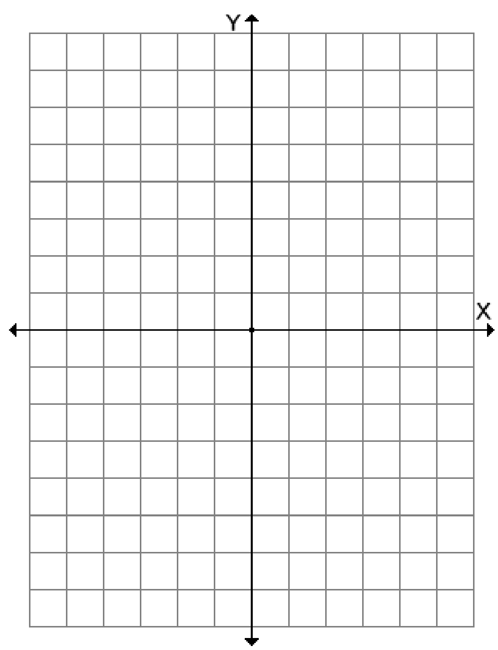
Range:

H.A:

EXAMPLE: Graph using transformations: $y = -\left(\frac{1}{2}\right)^x + 3$.

Indicate the domain and range. State the horizontal asymptote.





Logarithms

Exponential form: $x = b^y$ **Logarithmic form**: $y = \log_b x$

EXAMPLE: Change $\log_c 6 = 8$ into exponential form.

EXAMPLE: Change $2^d = 8$ into logarithmic form.

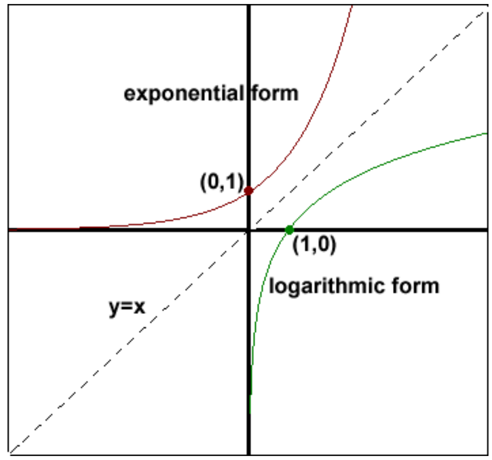
Equal Bases Property

If $a^u = a^v$ then u = v.

EXAMPLE: Find the exact value of $\log_4 64$.

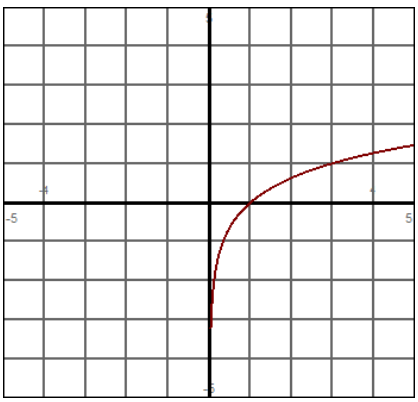
EXAMPLE: Find the exact value of $\log \frac{1}{10000}$.

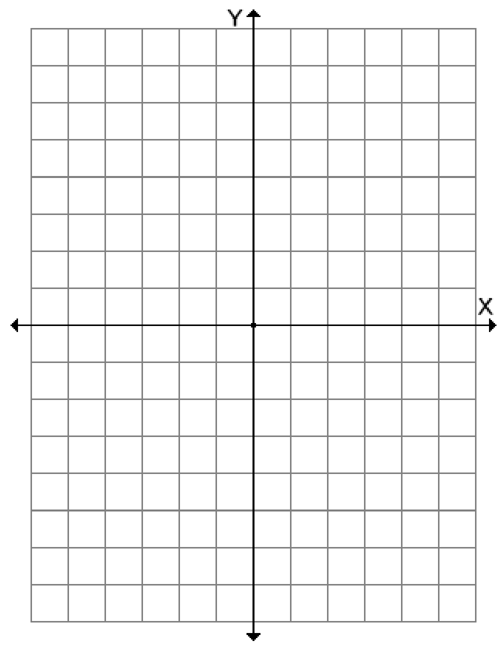
Let's try and draw a graph of $y = \log_b x$.



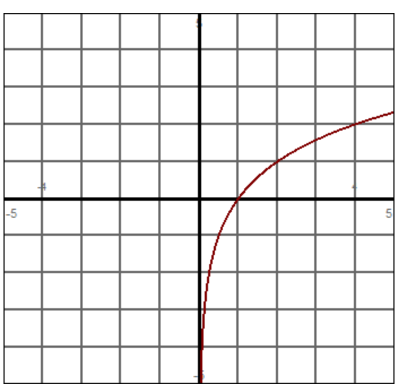
Key points: (1,0) and (b, 1).

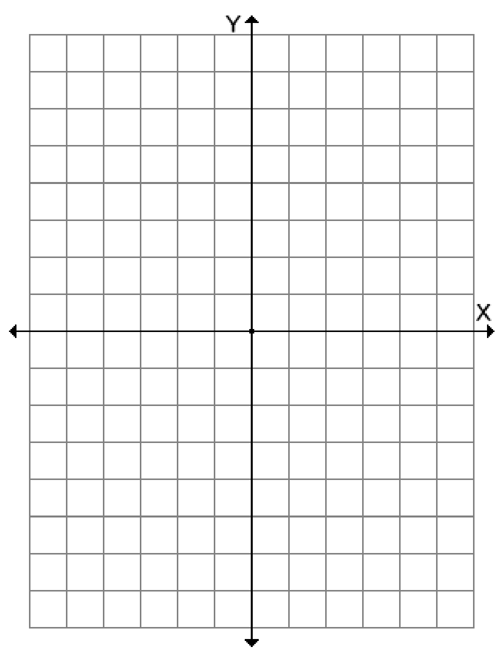
EXAMPLE: Graph using transformations: $y = \log_3(x-4)$.



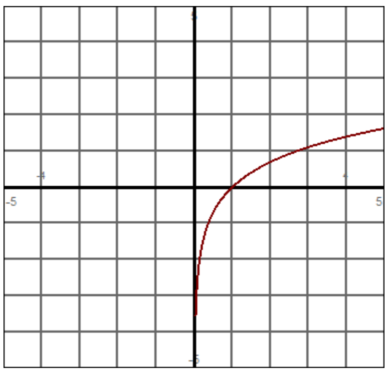


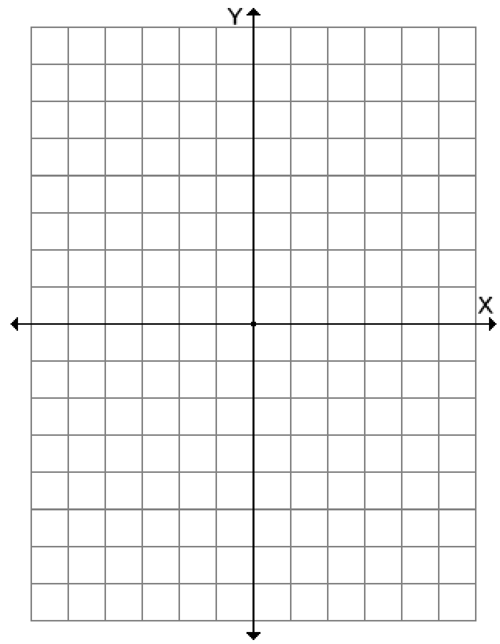
EXAMPLE: Graph using transformations: $y = -\log_2(x+3)$.





EXAMPLE: Graph using transformations: $y = \ln(1-x)$.





Algebraic Properties of Logarithms

For any numbers $x > 0$ and $y > 0$,

1.) *Product Rule*: $\log_a xy = \log_a x + \log_a y$

2.) *Quotient Rule*: $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

3.) *Reciprocal Rule*: $\log_a\left(\frac{1}{y}\right) = -\log_a y$

4.) *Power Rule*: $\log_a x^y = y\log_a x$

Inverse Properties for a^x and $\log_a x$

1.) Base a: $a^{\log_a x} = x$, $\log_a a^x = x$ $\quad a > 0,\ a \neq 1,\ x > 0$

2.) Base e: $e^{\ln x} = x$, $\ln e^x = x$ $\quad x > 0$

EXAMPLE: Find the exact value using logarithm properties: $\log_3\left(\frac{1}{3}\right)$.

EXAMPLE: Find the exact value using logarithm properties: $\log_{144} 12$.

EXAMPLE: Find the exact value using logarithm properties: $e^{\ln 6 - \ln 7}$.

EXAMPLE: Express $\log_9 x^2 \cdot \sqrt{3x-5}$ as a sum or difference of logarithms. Express powers as factors.

EXAMPLE: Express $\ln \frac{(x+5)^4}{x^3}$ as a sum or difference of logarithms. Express powers as factors.

EXAMPLE: Express $\log_4 \frac{(x-5)^5 \cdot \sqrt[3]{x-2}}{(x-1)^4}$ as a sum or difference of logarithms. Express powers as factors.

Solving Logarithmic and Exponential Equations

EXAMPLE: Solve: $4^{x-2} - 64 = 0$.

EXAMPLE: Solve: $3^{x} = 7$.

EXAMPLE: Solve: $e^{x+5} = 4$.

EXAMPLE: Solve: $\log_5(4x+5)=2$.

EXAMPLE: Solve: $\log_2(x+11)+\log_2(x+7)=5$

EXAMPLE: Solve: $\log_2(x+3)-\log_2(x+5)=1$