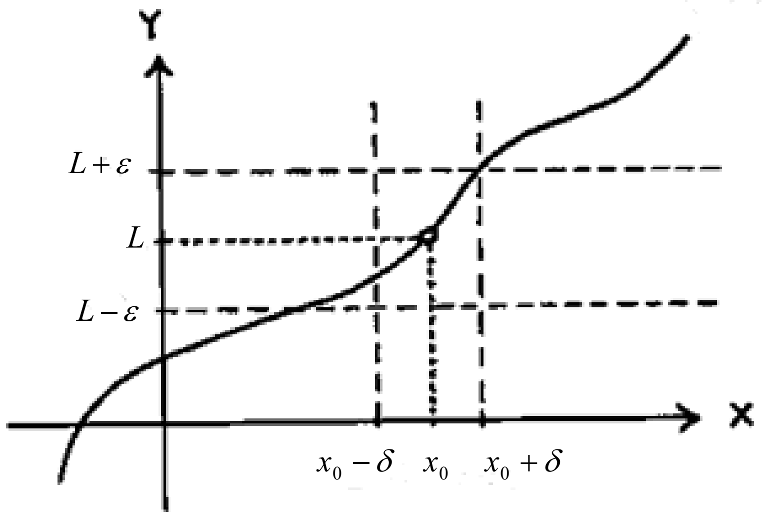
2.2 The Precise Definition of a Limit

Precise definition of a limit



Let *f* be defined on an open interval containing c and let L be a real number. Then:

$\lim_{x\to x_0} f(x)=L$ means that for each $\varepsilon>0$ there exists a $\delta>0$ such that if $0<|x-x_0|<\delta$, then $|f(x)-L|<\varepsilon$.

EXAMPLE: Use the ε - δ definition of a limit to prove that $\lim_{x\to 3} 5x-4=11$.

<u>Proof:</u>
For each $\varepsilon>0$ there exists a $\delta>0$ such that if $0<|x-3|<\delta$, then $|5x-4-11|<\varepsilon$.

EXAMPLE: Use the ε - δ definition of a limit to prove that $\lim_{x\to 4} \frac{x}{2}+6=8$.

<u>Proof:</u>

For each $\varepsilon > 0$ there exists a $\delta > 0$ such that if $0<|x-4|<\delta$, then $\left|\frac{x}{2}+6-8\right|<\varepsilon$.

EXAMPLE: In the following exercises, find an open interval about x_0 on which the inequality $|f(x)-L|<\varepsilon$ holds. Then give the largest value for $\delta > 0$ such that for all x satisfying $0<|x-x_0|<\delta$ the inequality $|f(x)-L|<\varepsilon$ holds:

a.) $f(x)=2x-2$, L = -6, $x_0=-2$, $\varepsilon = 0.02$

b.) $f(x) = \sqrt{x-7}$, L = 4, $x_0 = 23$, $\varepsilon = 1$

c.) $f(x) = 1/x$, L = -1, $x_0 = -1$, $\varepsilon = 0.1$

d.) $f(x) = x^2$, L = 3, $x_0 = \sqrt{3}$, $\varepsilon = 0.1$ (Round answers to four decimal places)