

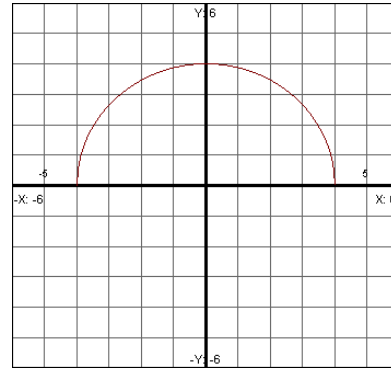
2.3 One-Sided Limits

One-Sided Limits

$\lim_{x \rightarrow c^+} f(x) = L$ This means we are finding the limit of f as we approach c from the right (positive side)

$\lim_{x \rightarrow c^-} f(x) = L$ This means we are finding the limit of f as we approach c from the left (negative side)

EXAMPLE: Find the limit: $\lim_{x \rightarrow 4^-} \sqrt{16 - x^2}$

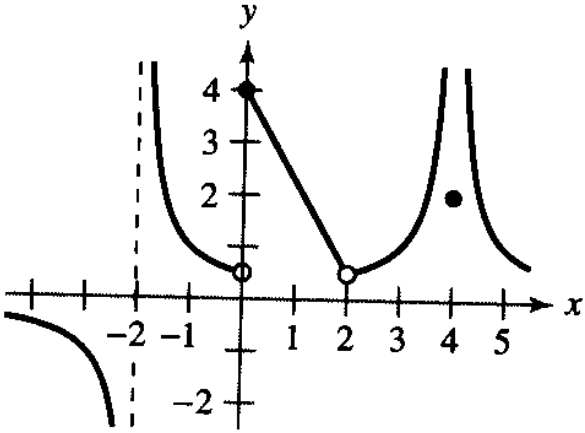


EXAMPLE: Find the limit: $\lim_{x \rightarrow 2^+} \frac{2 - x}{x^2 - 4}$

Finding One-Sided Limits Algebraically

EXAMPLE: Find the limit: $\lim_{x \rightarrow 1^-} \left(\frac{1}{x+1} \right) \left(\frac{x+6}{x} \right) \left(\frac{3-x}{7} \right)$.

EXAMPLE: Use the graph of $f(x)$ below to find the following:



a.) $f(0)$

b.) $f(2)$

c.) $f(-2)$

d.) $f(4)$

e.) $\lim_{x \rightarrow 2^+} f(x)$

f.) $\lim_{x \rightarrow 2^-} f(x)$

g.) $\lim_{x \rightarrow 2} f(x)$

h.) $\lim_{x \rightarrow 0^+} f(x)$

i.) $\lim_{x \rightarrow 0^-} f(x)$

j.) $\lim_{x \rightarrow 0} f(x)$

k.) $\lim_{x \rightarrow -2} f(x)$.

Limits involving $\frac{\sin \theta}{\theta}$.

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin \theta^2}{\theta^2} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin k\theta}{k\theta} = 1 \text{ where } k \text{ is any real number.}$$

EXAMPLE: Find $\lim_{\theta \rightarrow 0} \frac{\sin 8\theta}{5\theta}$.

EXAMPLE: Find $\lim_{\theta \rightarrow 0} \frac{\theta^2 - \theta + \sin \theta}{3\theta}$.

Limits involving $\frac{1 - \cos \theta}{\theta}$.

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta} = 0 \quad \text{where } \theta \text{ is in radians.}$$

EXAMPLE: Find $\lim_{\theta \rightarrow 0} \frac{1 - \cos 3\theta}{6\theta}$

EXAMPLE: Find $\lim_{x \rightarrow 0} \frac{\sin x - \sin x \cos 3x}{(3x^2 \sin^2 x + 3x^2 \cos^2 x)}$

EXAMPLE: Find $\lim_{x \rightarrow 0} 9x^2 \cot x \csc(3x)$