3.10 Related Rates

This section involves word problems dealing with how something is changing with respect to time. We will first find a formula that is used for the problem and then we will take the derivative of it with respect to time. Before we get to the part where we come up with the equation, let's first practice taking a derivative implicitly with respect to time.

EXAMPLE: If $V = 2(x^2 - 3y)$, find $\frac{dV}{dt}$ by using implicit differentiation. Then find $\frac{dV}{dt}$ when x = 3 given $\frac{dx}{dt} = -1$ and $\frac{dy}{dt} = 4$.

EXAMPLE: If $x^2 + y^3 = 15$, find $\frac{dy}{dt}$ when x = 3, y = 4, and $\frac{dx}{dt} = 8$.

EXAMPLE: A point is moving along the graph of $y = \frac{1}{1+x^2}$ such that the point's movement in the x direction is 2 cm/sec. Find how fast the point is moving in the y direction when x = -2?

EXAMPLE: A stone is dropped into a puddle causing ripples in the form of concentric circles. The radius, r, of the outer ripple is increasing at a rate of 2 cm/sec. When the radius is 0.5 meters, at what rate is the total area, A, of the disturbed water changing with respect to time? Note, the area of a circle is $A = \pi \cdot r^2$.

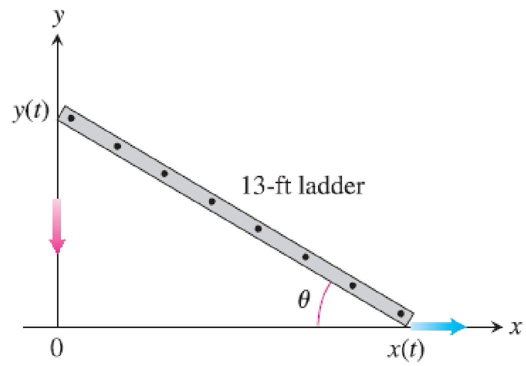
EXAMPLE: The volume of a sphere is $V = \frac{4}{3}\pi \cdot r^3$. If the radius is changing at a rate of 2 in/min, find the rate the volume is changing when the radius is 6 inches.

EXAMPLE: A conveyor belt drops sand into a conical pile whose radius is always twice the height. Sand falls at a rate of 5 cubic meters per second. How fast is its height increasing when the radius is 20 meters?

Note: you will need to use the volume of cone formula, which is $V = \frac{1}{3}\pi \cdot r^2 h$.

EXAMPLE: The length L of a rectangle is decreasing at the rate of 2 cm/sec while the width W is increasing at the rate of 2 cm/sec. When L = 12 cm and W = 5 cm, find the rates of change of **a)** the area, **b.)** the perimeter, and **c.)** the lengths of the diagonals of the rectangle. Which of these quantities are decreasing, and which are increasing?

EXAMPLE: A 13-foot ladder is leaning against a house when its base starts to slide away as shown below. By the time the base is 12 feet from the house, the base is moving at a rate of 5 ft/sec. **a.)** Find the rate at which the ladder is falling down the side of the house. **b.)** Consider the triangle formed by the ladder, house, and ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 12 feet from the base of the house. **c.)** At what rate is the angle θ changing when the base of the ladder is 12 feet from the base of the house?



EXAMPLE: A police cruiser, approaching a right-angled intersection from the north is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 miles north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60mph at the instant of measurement, what is the speed of the car?

EXAMPLE: A hot air balloon rising straight up from a level field is tracked by a range finder 500 feet from the liftoff point. At the moment the range finder's elevation angle is $\frac{\pi}{4}$ the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at that moment?

EXAMPLE: A man 6 ft tall walks at the rate of 3 ft/s toward a streetlight that is 15 ft above the ground. **(a)** At what rate is the tip of his shadow moving? **(b)** At what rate is the length of his shadow decreasing when he is 40 ft from the base of the light?