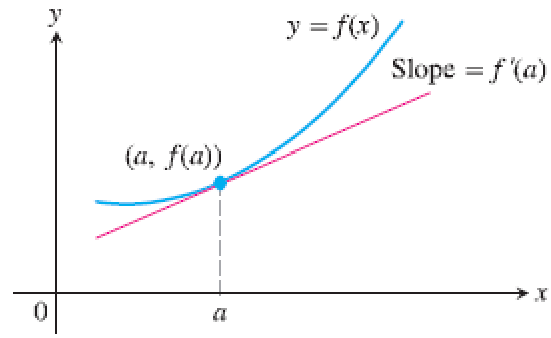
3.11 Linear Approximations and Differentials

Sometimes we can approximate more complicated functions with simpler ones. These would give us enough accuracy for certain situations and are easier to work with. The approximating functions in this section are called **linearizations**, and they are based on tangent lines. It is possible to have approximating functions that are polynomials, but we will not discuss this here.



EXAMPLE: Find the linearization $L(x)$ of $f(x)=\sqrt{x^2+16}$ at $x=-3$.

EXAMPLE: Find the linearization $L(x)$ of $f(x) = \sin x$ at $x = \dfrac{\pi}{6}$.

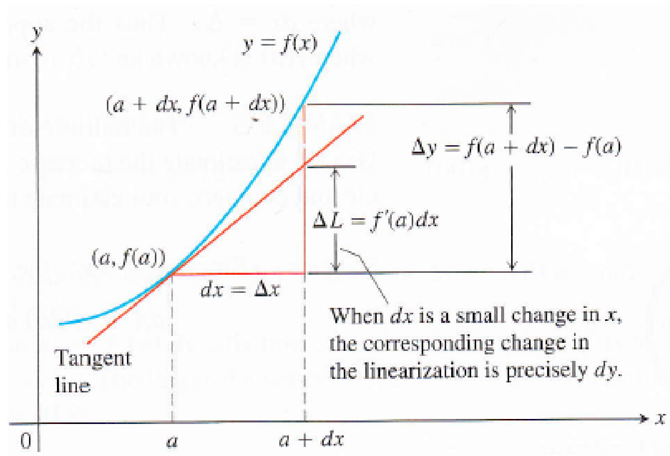
EXAMPLE: Given $x_0 = -0.9$, find a linearization that will replace $f(x) = 3x^2 + 2x - 4$ by centering your linearization not on x_0, but at a nearby integer $x = a$. This will make the derivative and function easy to evaluate.

EXAMPLE: Given $x_0 = \dfrac{11}{5}$, find a linearization that will replace $f(x) = \dfrac{2x}{x-1}$ by centering your linearization not on x_0, but at a nearby integer $x = a$. This will make the derivative and function easy to evaluate.

EXAMPLE: Given $x_0 = \dfrac{\pi}{12}$, find a linearization that will replace $f(x) = \tan^{-1} x$ by centering your linearization not on x_0, but at a nearby integer $x = a$. This will make the derivative and function easy to evaluate.

Differentials

Back in the section regarding implicit differentiation, we were solving for $\frac{dy}{dx}$. We treated this as one variable. However in this section we will consider this made up of two different variables, *dy* and *dx*. Let $y = f(x)$ be a differentiable function. The **differential *dx*** is an independent variable. The **differential *dy*** is $dy = f'(x)dx$. But what does this mean in terms of something physical? The below diagram illustrates this geometrically.



EXAMPLE: Find *dy* given $y = x^4 - 6\sqrt{x}$.

EXAMPLE: Find *dy* given $xy^3 - 8x^{\frac{3}{2}} - y = 0$.

EXAMPLE: Estimate the volume of material in a cone-shaped shell with a height of 20 in, radius 9 in, and shell thickness 0.5 in.

EXAMPLE: The radius of a sphere is measured as 6 cm with an error of 3%. The volume of the sphere is to be calculated from this measurement. Estimate the percentage error in the volume calculation.