3.3 Differentiation Rules

The derivative has several different types of notation:

Notation	Meaning
$f'(x)$	Derivative of $f(x)$
$\frac{dy}{dx}$	Derivative of y with respect to x.
y'	Derivative of y
$\frac{d}{dx}[f(x)]$	Derivative of f with respect to x.

Constant Rule

$\frac{d}{dx}[c]=0$ This means the derivative of any number is zero. For example, suppose we had $y=9$ and the question asked us to find y'. Then our answer would automatically be zero because y = 9 is a horizontal line. The slope of a horizontal line is always 0.

EXAMPLE: Find $f'(x)$ if $f(x)=4\cdot\pi\cdot e^2$.

Can you see a pattern of the derivative with the following table?

Original function	Derivative
$y=x^2$	$y'=2x$
$y=x^3$	$y'=3x^2$
$y=x^5$	$y'=5x^4$

Power Rule

If n is any real number, then $\frac{d}{dx}x^n = nx^{n-1}$ for all x where powers of x^n and x^{n-1} are defined.

Other Derivative Rules

$$\frac{d}{dx}\left[f(x)+g(x)\right]=f'(x)+g'(x)$$
$$\frac{d}{dx}\left[f(x)-g(x)\right]=f'(x)-g'(x)$$
$$\frac{d}{dx}\left[c\cdot f(x)\right]=c\cdot f'(x)$$

EXAMPLE: If $y=x^{12}$, find y'.

EXAMPLE: If $y=3x^4$, find y'.

EXAMPLE: If $g(x)=\frac{3}{2}x^6-x+3$, find $g'(x)$.

EXAMPLE: If $y=3x(6x-5x^2)$, find y'.

EXAMPLE: If $f(x)=\sqrt[4]{x}$, find $f'(x)$.

EXAMPLE: If $f(x)=\dfrac{2}{x^3}$, find $f'(x)$.

EXAMPLE: If $f(x)=x+\dfrac{1}{x}$, find $f'(x)$.

EXAMPLE: If $f(x)=\dfrac{2x^2-3x+1}{x}$, find $f'(x)$.

EXAMPLE: If $y=\sqrt{x}-\dfrac{5}{\sqrt{x}}+\sqrt{2}$, find y'.

EXAMPLE: Determine the point(s) at which $y = x^2 + 1$ has a horizontal tangent line.

EXAMPLE: Determine the point(s) at which $y = x^3 - 27x$ has a horizontal tangent line.

Product Rule

$$\frac{d}{dx}[f \cdot g] = f \cdot g' + g \cdot f'$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{g \cdot f' - f \cdot g'}{g^2}$$

EXAMPLE: Use the product rule to find $H'(x)$ if $H(x) = (6x+5)(x^3 - 2)$.

EXAMPLE: Use the product rule to find $M'(x)$ if $M(x)=\sqrt{x}\left(4-x^2\right)$. Write answer as a single fraction.

EXAMPLE: Use the quotient rule to find $H'(x)$ if $H(x)=\dfrac{x}{\sqrt{x}-1}$.

Derivation of the derivative of e^x

EXAMPLE: Given: $y = e^{-x}$ find y'.

EXAMPLE: Given: $y = \frac{e^x - e^{-x}}{2}$ find y'.

EXAMPLE: Given: $y = x^2 e^x$ find y'.

EXAMPLE: Given: $y = \left(9x^2 - 6x + 2\right)e^x$ find y'.

EXAMPLE: Given: $y = \sqrt[11]{x^3} - e^3 + x^e$ find y'.

EXAMPLE: Given: $y = \dfrac{4e^x}{2x^5 - 3e^x}$ find y'.

Higher Order Derivatives

$f(x)$	This is our original function
$f'(x)$	First derivative of f
$f''(x)$	Second derivative of f (derivative of $f'(x)$)
$f'''(x)$	Third derivative of f (derivative of $f''(x)$)
$f^{(n)}(x)$	The nth derivative of f (derivative of $f^{(n-1)}(x)$)

A function has derivatives of all orders once the derivative reaches zero.

EXAMPLE: Let $f(x) = 4x^3 + 5x^2 + 3x + 1$. Find the derivatives of all orders.

EXAMPLE: Let $f(x) = \dfrac{x^3 + 2x^2 - 1}{x}$. Find $f'''(x)$.

EXAMPLE: Given: $y = \left(\frac{x^3 - 2}{5x}\right)\left(\frac{x^2 + 5}{x^3}\right)$ find y' and y''.

EXAMPLE: Given: $y = 4x^3e^{-x}$ find y' and y''.