

3.6 The Chain Rule

Sometimes we need to take the derivative of an expression raised to a large power, and it would take too long to multiply all of it out and use the power rule. The chain rule allows us to be able to do this. We will start with $y = f(u)$. The outside function is f and the inside function will be u . The chain rule says the following:

Chain Rule

If $y = f(u)$ then $y' = f'(u) \cdot u'$ Which is also the same as $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ just using different notation.

EXAMPLE: Write the expression in the form $y = f(u)$ and $u = g(x)$ Then find $\frac{dy}{dx}$ if $y = \frac{-5}{(x+3)^3}$.

EXAMPLE: Write the expression in the form $y = f(u)$ and $u = g(x)$ Then find $\frac{dy}{dx}$ if $y = \left(\frac{1}{6x} - \frac{x}{6}\right)^6$.

EXAMPLE: Write the expression in the form $y = f(u)$ and $u = g(x)$ Then find $\frac{dy}{dx}$ if $y = \csc(\cot \theta)$.

EXAMPLE: Write the expression in the form $y = f(u)$ and $u = g(x)$ Then find $\frac{dy}{dx}$ if $y = (9x - 7)^4$.

EXAMPLE: Write the expression in the form $y = f(u)$ and $u = g(x)$ Then find $\frac{dy}{dx}$ if $y = \sqrt{3x^2 - 4x + 6}$.

EXAMPLE: Write the expression in the form $y = f(u)$ and $u = g(x)$ Then find $\frac{dy}{dx}$ if $y = \sin^5 x$.

EXAMPLE: Find the derivative of the function: $y = \sqrt[6]{x^7 - 5x^4}$.

EXAMPLE: Find y' if $y = x\sqrt{4 + x^2}$.

EXAMPLE: Find y' if $y = \frac{(6x+1)^4}{(2x+1)^5}$.

EXAMPLE: Find $h'(t)$ if $h(t) = \left(\frac{t^2}{t^3 + 2} \right)^2$. Write your answer as a single fraction.

EXAMPLE: Find $f'(\theta)$ if $f(\theta) = \sin(\cos(9\theta))$.

EXAMPLE: Find y' if $y = \cos\left[(4x + 7)^{\frac{3}{4}}\right]$.

EXAMPLE: Find y' if $y = 2 \cot^2(\pi x + 2)$.

EXAMPLE: Find y' if $y = e^{-x^2}$.

EXAMPLE: Find y' if $y = \left(e^{\sin(x/2)}\right)^6$.

EXAMPLE: Find the derivative of $y = \sin\left(\frac{\theta}{\sqrt{\theta-4}}\right)$.