

3.8 Derivatives of Inverse and Logarithmic Functions

Derivatives of Inverses of Differentiable Functions

EXAMPLE: Given $f(x) = x^2 - 1$ and $f^{-1}(x) = \sqrt{x+1}$, find $\frac{d}{dx} f(x)$ and $\frac{d}{dx} f^{-1}(x)$.

Derivative Rule for Inverses

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

EXAMPLE: Given $f(x) = \sqrt{x+7}$ and $f^{-1}(x) = x^2 - 7$, evaluate $\frac{d}{dx} f(x)$ at $x = 2$ and $\frac{d}{dx} f^{-1}(x)$ at $x = f(2)$

EXAMPLE: Let $f(x) = 3x^2 - 7x + 2$. Find the value of $\frac{d}{dx} f^{-1}(x)$ at $x = 0 = f(2)$.

Derivative of a Natural Logarithm

Let u be a differentiable function of x . Then:

1.) $\frac{d}{dx} [\ln x] = \frac{1}{x}$ where $x > 0$

2.) $\frac{d}{dx} [\ln u] = \frac{1}{u} \cdot \frac{du}{dx} = \frac{u'}{u}$ where $u > 0$

EXAMPLE: Find the derivative: $y = \ln\left(\frac{9}{x^2}\right)$.

EXAMPLE: Find the derivative: $f(x) = \ln|\csc x|$.

EXAMPLE: Find the derivative: $y = \frac{1 - \ln x}{x}$.

EXAMPLE: Find the derivative: $y = \ln(\sin(\ln \theta))$.

EXAMPLE: Find the derivative: $y = \ln\left(\frac{2x}{x+3}\right)$.

EXAMPLE: Find the derivative: $y = \ln\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}$.

EXAMPLE: Find the derivative: $y = \ln\sqrt{\frac{\sin\theta\cos\theta}{1-4\ln\theta}}$.

EXAMPLE: Find the derivative: $y = \ln\left(\frac{1+e^x}{1-e^x}\right)$.

Logarithmic Differentiation

The process of logarithmic differentiation involves taking the derivative of both sides of the equation.

EXAMPLE: Use logarithmic differentiation to find the derivative of: $y = \sqrt[3]{x(x-4)}$.

EXAMPLE: Use logarithmic differentiation to find the derivative of: $y = (x - 2)^{x+1}$.

EXAMPLE: Use logarithmic differentiation to find the derivative of: $y = \frac{\theta \sin \theta}{\sqrt{\sec^3 \theta}}$.

Derivation of derivative of a^x

Derivation of the derivative of $\log_a x$

EXAMPLE: Find the derivative: $y = 8^{x^2}$.

EXAMPLE: Find the derivative: $y = x(6^{-2x})$.

EXAMPLE: Find the derivative: $y = \log_4(2 + x \ln 4)$.

EXAMPLE: Find the derivative: $y = \log_3 \frac{x\sqrt{x-1}}{2}$. Write your answer as a single fraction.

EXAMPLE: Find the derivative: $y = \log\left(\frac{\sin x \sec x}{e^x \cdot 3^x}\right)^{\ln 10}$. Write your answer as a single fraction.

EXAMPLE: Find the derivative: $y = \theta \cdot \log_2\left(e^{(\cos \theta)(\ln 2)}\right)$.