

3.9 Derivatives of Inverse Trigonometric Functions

EXAMPLE: Write in algebraic form: $\sec(\tan^{-1} 4x)$.

EXAMPLE: Write in algebraic form: $\cot(\sin^{-1}(x-1))$.

Derivative of inverse trig functions

EXAMPLE: If $y = \sin^{-1} u$, find y' .

EXAMPLE: If $y = \sec^{-1} u$, find y' .

Derivatives of Inverse Trig Functions

$$\frac{d}{dx} [\sin^{-1} u] = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\cos^{-1} u] = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\tan^{-1} u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx} [\cot^{-1} u] = -\frac{u'}{1+u^2}$$

$$\frac{d}{dx} [\sec^{-1} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} [\csc^{-1} u] = -\frac{u'}{|u|\sqrt{u^2-1}}$$

EXAMPLE: Find the derivative: $y = \sec^{-1}(2x)$.

EXAMPLE: Find the derivative: $y = \cos^{-1}(5x^4)$.

EXAMPLE: Find the derivative: $y = \cot^{-1}(3x^2 - 8)$.

EXAMPLE: Find the derivative: $y = \tan^{-1} \sqrt{5-2x}$.

EXAMPLE: Find the derivative: $y = \ln(\cot^{-1} 2x^3)$.

EXAMPLE: Find the derivative: $y = x \cdot \tan^{-1}(2x) - \frac{1}{4} \ln(1 + 4x^2)$.

EXAMPLE: Find the derivative: $y = 25 \sin^{-1}\left(\frac{x}{5}\right) - x\sqrt{25 - x^2}$. Write your answer as a single fraction.