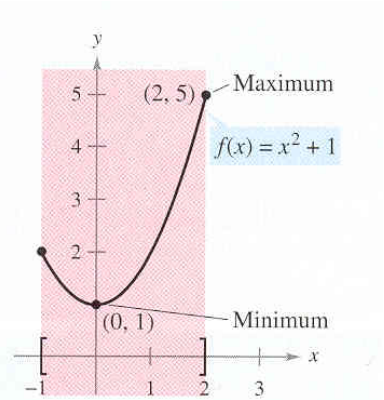
4.1 Maxima and Minima

Extrema of a Function

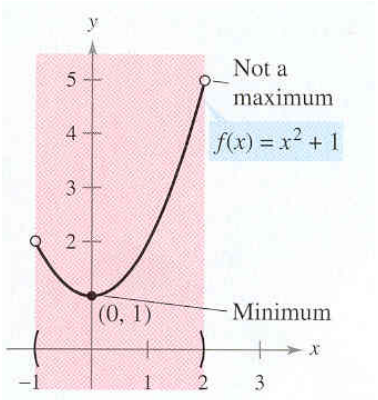
1.) $f(c)$ is the minimum of f on interval I if $f(c) \le f(x)$ for all x in I.

2.) $f(c)$ is the maximum of f on interval I if $f(c) \ge f(x)$ for all x in I.

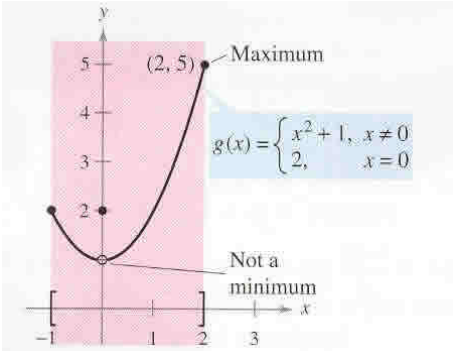
A function is guaranteed to have a minimum and maximum value if the function is continuous on a closed interval. The following are examples of when a function may or may not have a min or max:



(a) f is continuous, $[-1, 2]$ is closed.



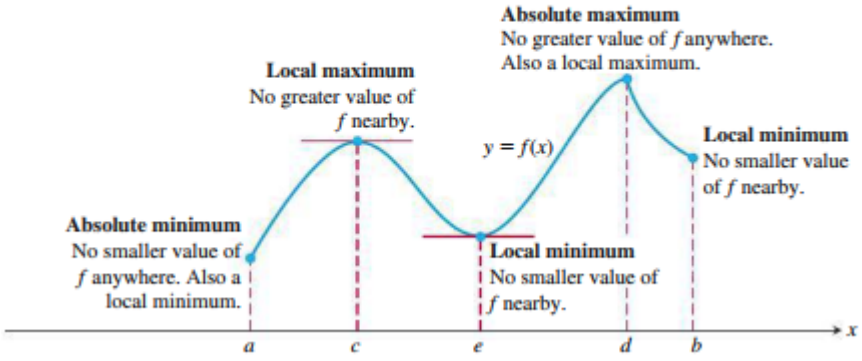
(b) f is continuous, $(-1, 2)$ is open.



(c) g is not continuous, $[-1, 2]$ is closed. Extrema can occur at interior points or endpoints of an interval. Extrema that occur at the endpoints are called **endpoint extrema.**

Local Extreme Values

Local extreme values means there is a hill or valley in the graph. This is not necessarily the absolute maximum or minimum of the graph. In the picture to the right we see a hill and valley but they are not the highest and lowest point of the graph. The highest or lowest possible points on the graph are called **absolute extrema**, and these are labeled on the graph.



Critical Number

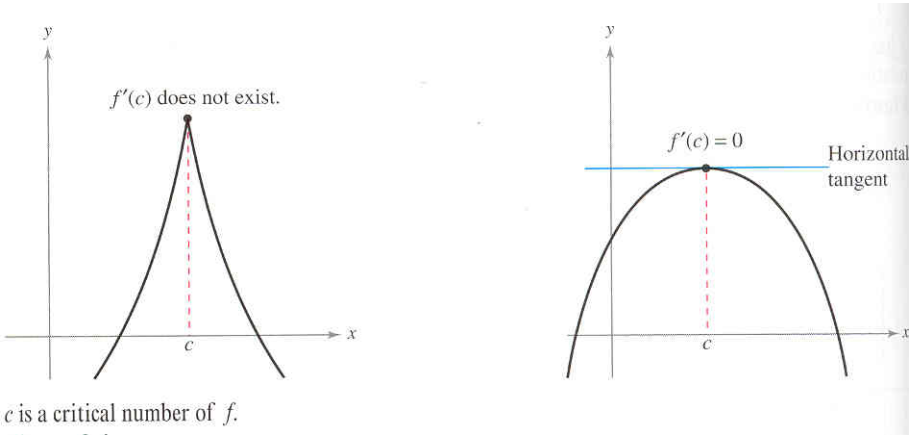
Let f be defined at c. The point x = c is a critical number if **either** of the following occurs which is shown in the pictures below:

1.) $f'(c)$ is undefined <u>OR</u>

2.) $f'(c) = 0$

<u>AND</u>

$f(c)$ is defined.



c is a critical number of *f*.

EXAMPLE: Find the derivative of $y = |x|$ at the point (0, 0) by using the limit process.

How to find extrema on a closed interval [a, b]

1.) Take the first derivative and set it equal to zero to find the critical point. You may also need to see if the derivative is undefined anywhere in the interval, because this will also give you a critical point.

2.) Evaluate *f* at each critical point you find in the interval.

3.) Evaluate *f* at the endpoints of your interval. (In other words, find $f(a)$ and $f(b)$).

4.) The least of these values is the absolute minimum. The greatest is the absolute maximum.

EXAMPLE: Let $f(x) = 3x^4 + 4x^3 - 12x^2$ on [-4, 2]. Find all critical numbers and absolute extrema on this interval.

EXAMPLE: Let $f(x) = 12 \cdot \sqrt[3]{x^2} - 8x$ on [-1, 2]. Find all critical numbers and absolute extrema on this interval.

EXAMPLE: Let $f(\theta) = \sec\theta$ on $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$. Find all critical numbers and absolute extrema on this interval.

EXAMPLE: Let $f(x) = e^{-x^2}$ on [-2, 1]. Find all critical numbers. Then find the absolute extrema on this interval.

EXAMPLE: Let $f(x) = \frac{2x+5}{3}$ on [-1, 5]. Find all critical numbers. Then find the absolute extrema on this interval.

EXAMPLE: Find the extreme values (absolute and local) of the function over its natural domain, and where they occur: $f(x) = \dfrac{2}{x^2 - 1}$.

EXAMPLE: Find the extreme values (absolute and local) of the function over its natural domain, and where they occur: $f(x) = \cos^{-1}\left(e^{x}\right)$.