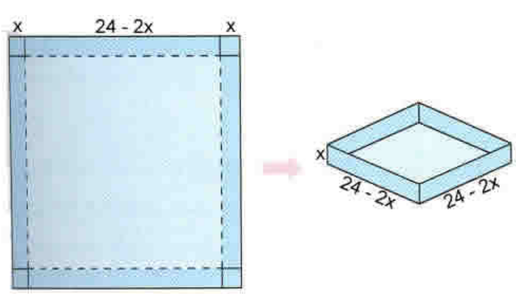
4.6 Applied Optimization Problems

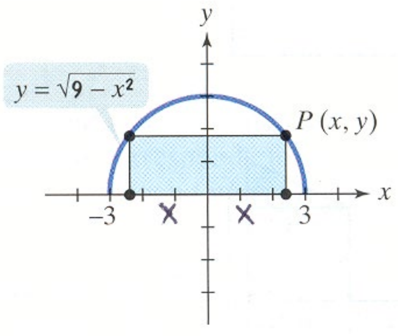
In this section we will be looking at word problems where it asks us to maximize or minimize something. For all the problems in this section you will be taking the derivative of something and setting it equal to zero.

EXAMPLE: What is the smallest perimeter possible for a rectangle whose area is 36 square inches, and what are the dimensions?

EXAMPLE: An open box of maximum volume is to be made from a square piece of material, 24 inches on a side, by cutting equal squares from the corners and turning up the sides (see figure). First write an equation for the volume, V, of the box as a function of x. Then find the maximum volume of the box.

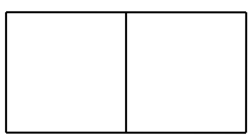


EXAMPLE: A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{9 - x^2}$ (see figure). What length and width should the rectangle have so that its area is a maximum? What is the maximum area?

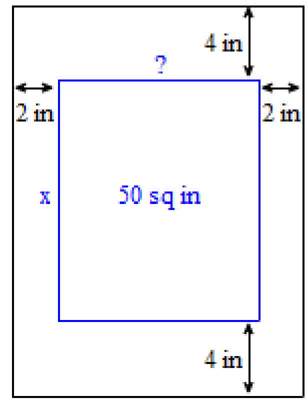


EXAMPLE: Two sides of a triangle have lengths a and b, and the angle between them is θ. What value of θ will maximize the triangle's area? (Hint: $A = \frac{1}{2}ab\sin\theta$)

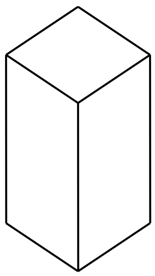
EXAMPLE: A rancher has 200 feet of fencing which to enclose two adjacent corrals. What dimensions should be used so that the enclosed area will be a maximum? (See figure)



EXAMPLE: You are designing a poster to contain 50 square inches of printing with a 4 inch margin at the top and bottom and a 2 inch margin at each side (see figure). What overall dimensions will minimize the amount of paper used?



EXAMPLE: A 108 cubic foot box with a square base and an open top (see figure) is to be constructed from sheet metal of a given thickness. Find the dimensions of the tank with minimum weight.



EXAMPLE: You have been asked to design a 2000 cubic centimeter can shaped like a right circular cylinder. What dimensions will use the least material. NOTE: The surface area formula for a right circular cylinder is $S = 2\pi r^2 + 2\pi rh$. The area of a right circular cylinder is $A = \pi r^2 h$.

EXAMPLE: The height above ground of an object moving vertically is given by $s = -16t^2 + 96t + 112$ with s in feet and t in seconds. Find **a.)** the object's velocity when t = 0, **b.)** its maximum height and when it occurs, and **c.)** its velocity when s = 0.