

5.3 The Definite Integral

Let $\|P\|$ be the norm, which represents the largest subinterval (partition), or the largest width.

$$\|P\| = \Delta x = \frac{b-a}{n}. \quad \text{As } \|P\| \rightarrow 0, \quad n \rightarrow \infty.$$

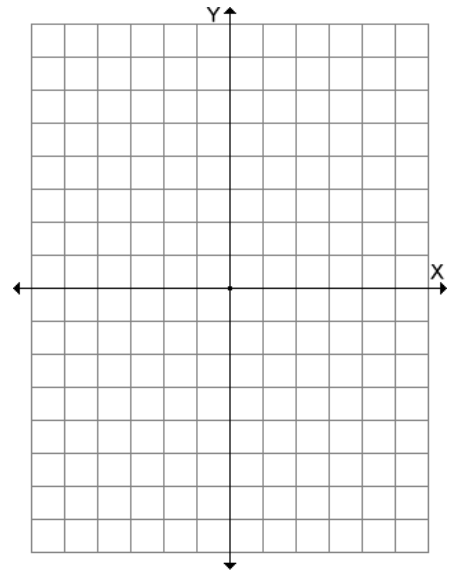
$$\lim_{\|P\| \rightarrow 0} \sum_{i=1}^n f(c_i) \Delta x_i = \int_a^b f(x) dx.$$

EXAMPLE: Evaluate using the limit process: $\int_{-2}^3 x \, dx$.

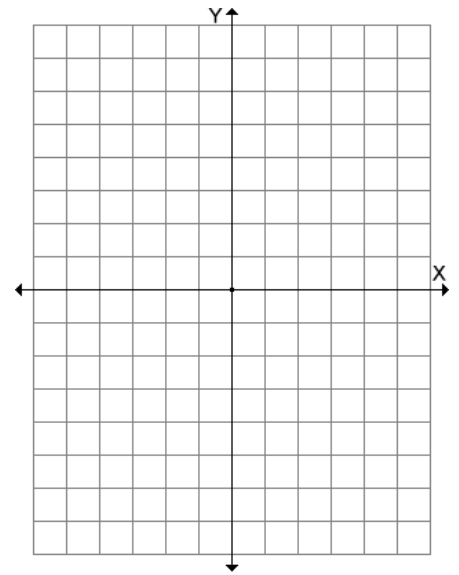
EXAMPLE: Change the following into a definite integral: $\lim_{\|P\| \rightarrow 0} \sum_{i=1}^n 6c_i (4 - c_i)^2 \Delta x_i$ where P is a partition of $[0, 4]$.

EXAMPLE: Change the following into a definite integral: $\lim_{\|P\| \rightarrow 0} \sum_{i=1}^n \left(\frac{3}{c_i^2} \right) \Delta x_i$ where P is a partition of $[1, 3]$.

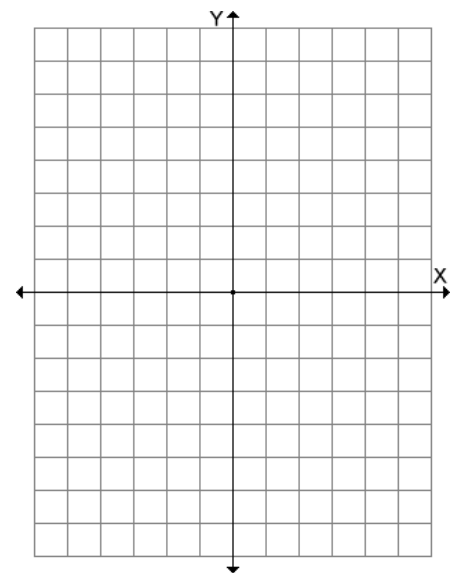
EXAMPLE: Graph the integrand $\int_0^2 -2x + 4 \, dx$, and use areas to evaluate the integral.



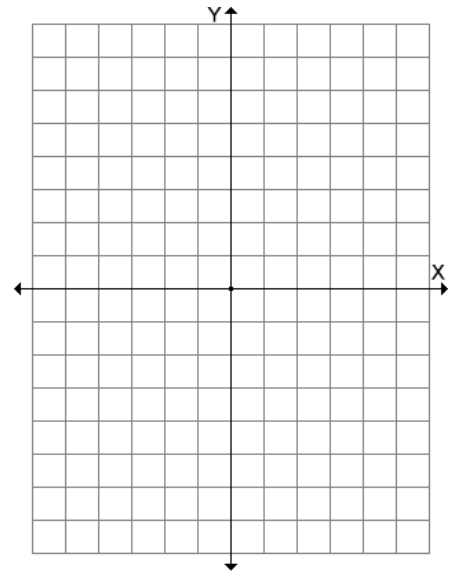
EXAMPLE: Graph the integrand $\int_{-3}^3 \sqrt{9-x^2} dx$, and use areas to evaluate the integral.



EXAMPLE: Graph the integrand $\int_0^2 x+1 dx$, and use areas to evaluate the integral.



EXAMPLE: Graph the integrand $\int_{-1}^3 2 - |x - 1| dx$, and use areas to evaluate the integral.



Properties of Definite Integrals (Assume f and g are integrable on $[a, b]$).

$$1.) \int_a^a f(x) dx = 0$$

From a to a we have a rectangle with a width of 0, so the area is 0.

$$2.) \int_a^b f(x) dx = -\int_b^a f(x) dx$$

If we switch the order of a and b the area changes sign.

$$3.) \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

We can split up the area in to two separate areas.

$$4.) \int_a^b k \cdot f(x) dx = k \int_a^b f(x) dx$$

We are allowed to take a constant k out of the integral.

$$5.) \int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

We can do two separate integrals.

EXAMPLE: Given $\int_2^4 x^3 dx = 60$, $\int_2^4 x dx = 6$, and $\int_2^4 dx = 2$ evaluate the following using properties of definite integrals:

a.) $\int_2^2 x^3 dx$

b.) $\int_2^4 15 dx$

c.) $\int_2^4 (x^3 + 4) dx$

d.) $\int_4^2 x(2 - x^2) dx$

EXAMPLE: Given $\int_1^3 f(x) dx = -2$, $\int_1^6 f(x) dx = 5$, and $\int_1^6 g(x) dx = 7$, evaluate the following using properties of definite integrals:

a.) $\int_3^3 g(x) dx$

b.) $\int_6^1 3f(x) dx$

c.) $\int_3^6 f(x) dx$

d.) $\int_6^1 [f(x) - g(x)] dx$

e.) $\int_1^6 [-2f(x) + g(x)] dx$