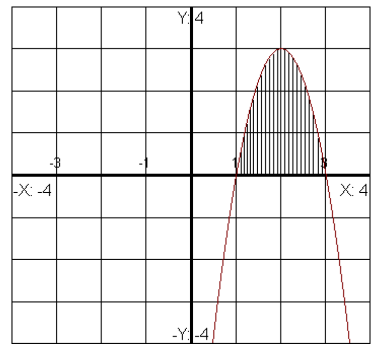
5.4 The Fundamental Theorem of Calculus

First Fundamental Theorem of Calculus

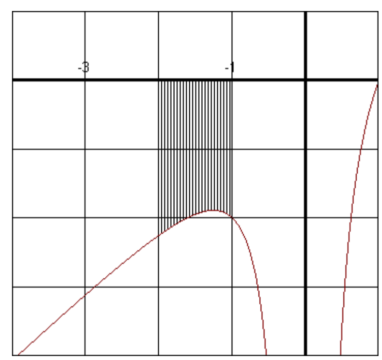
If a function f is continuous on the closed interval [a, b] and F is antiderivative of f on the interval [a, b] then

$$\int_a^b f(x)\ dx = F(b) - F(a)$$

EXAMPLE: Evaluate: $\int_1^3 -3x^2 + 12x - 9\ dx$.

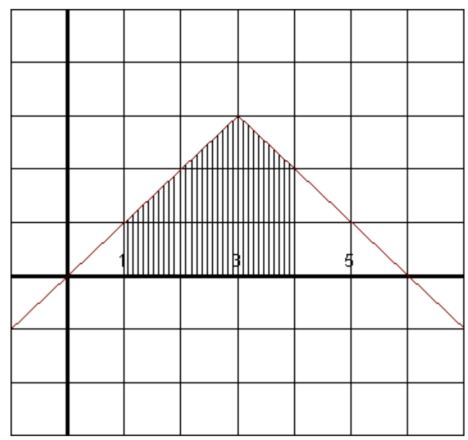


EXAMPLE: Evaluate: $\int_{-2}^{-1} u - \frac{1}{u^2}\ du$.



EXAMPLE: Evaluate: $\int_1^8 \frac{3x^2 - \sqrt[3]{x^2}}{3x^2}\ dx$.

EXAMPLE: Evaluate: $\int_1^4 3 - |x-3|\ dx$.



EXAMPLE: Evaluate: $\int_{0}^{\frac{\pi}{4}} \frac{1-\sin^2\theta}{\cos^2\theta}\,d\theta$.

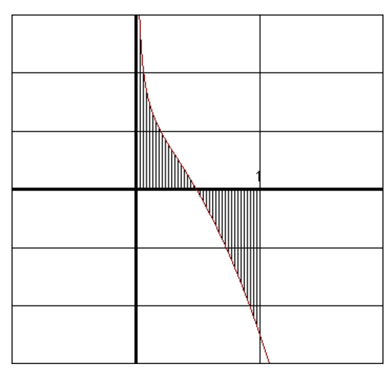
EXAMPLE: Evaluate: $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2-\csc^2\theta\ d\theta$.

EXAMPLE: Evaluate: $\int_{1}^{2} \frac{1}{x}-2e^{2x}\ dx$.

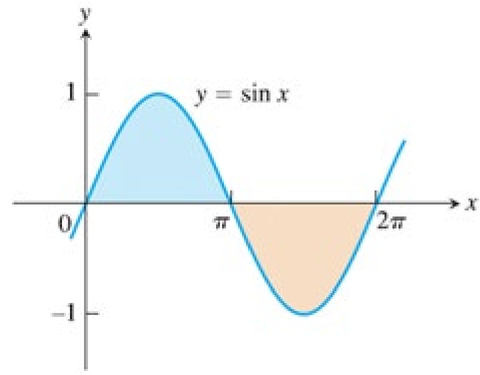
EXAMPLE: Evaluate: $\displaystyle\int_{\frac{1}{2}}^{\frac{1}{\sqrt{3}}} \frac{1}{x\sqrt{4x^2-1}}\,dx$.

EXAMPLE: Evaluate: $\displaystyle\int_{-1}^{2} 2^x\,dx$.

EXAMPLE: Evaluate: $\displaystyle\int_{0}^{1} \frac{1}{2\sqrt{x}} - 3x^2\,dx$



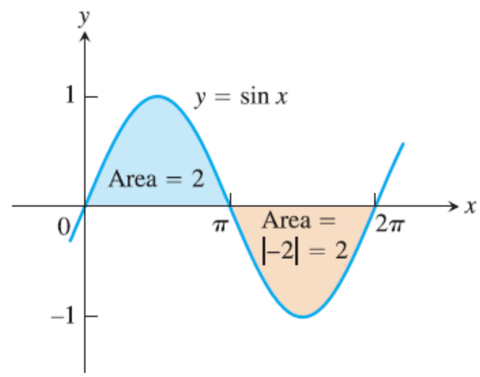
EXAMPLE: Evaluate: $\int_{0}^{2\pi} \sin x \; dx$



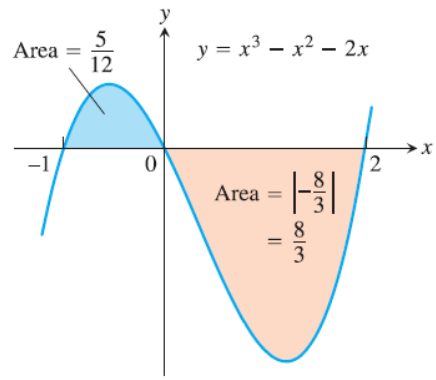
To find the total area between the graph of $y = f(x)$ and the x-axis over the interval [a, b]:

1.) Subdivide [a, b] at the zeros of *f*.
2.) Integrate f over each subinterval.
3.) Add the absolute values of the integrals.

EXAMPLE: Find the total area between the graph of $y = \sin x$ and the x-axis over the interval $[0, 2\pi]$.



EXAMPLE: Find the total area between the graph of $y = x^3 - x^2 - 2x$ and the x-axis if $-1 \le x \le 2$.



EXAMPLE: Find: $\frac{d}{dx}\int_4^x \sin\theta \; d\theta$.

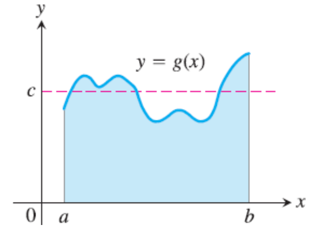
Second Fundamental Theorem of Calculus

Assume f is continuous on an open interval I containing a, then, for every x in the interval, the following is true:
$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\ dt\right] = f(g(x)) \cdot g'(x)$. Notice the value of a does not affect our answer.

EXAMPLE: Find $f'(x)$ if $f(x) = \int_{-1}^{x} \frac{t^2}{t^2+1} dt$.

EXAMPLE: Find $f'(x)$ if $f(x) = \int_{0}^{x^2} \sin\theta^2\ d\theta$.

Average Value of a Continuous Function



If you want to take an average of a list of items you would add them all up and then divide by number of items. Suppose you wanted to find the average value of a continuous function *f* on an interval [*a*, *b*]. The graph to the right illustrates this idea. The graph of *g*(*x*) has an average height of *c* between *a* and *b*. So geometrically the average (mean) value of *g*(*x*) on [a, b] is the area under the graph divided by *b* – *a*. The area under a graph can be represented with a definite integral. So we will take the definite integral and divide by *b* – *a*.

Here is the notation: av(*f*) = $\frac{1}{b-a}\int_a^b f(x)\ dx$.

EXAMPLE: Graph $f(x) = -3x^2 - 1$ and find its average value over the interval $[0, 1]$.