1.1 Review of Functions

Before we jump in calculus, we will first do a quick review of functions, graphs, and equations

Function Definition: For each input (x) there can only be one output (y).

Function notation: $f(x)$ which means "f of x". This does not mean f times x. It means that we have a function called f which contains the variable x.

EXAMPLE: Given the function $f(x) = 2x - 5$, find the following:

a.) Find $f(3)$. Solve $f(x) = 7$.

Whatever is inside the parenthesis goes in place of x in the original expression. This is really asking us for the y value when x is 3.

$f(3) = 2(3) - 5$
$f(3) = 1$

To solve $f(x) = 7$, we put in a 7 for $f(x)$: $7 = 2x - 5$. Now we solve for x. Add the 5 to both sides and divide both sides by 2 to get the answer of 6.

b.) $f(x+3)$

Now we need to replace x in the original equation with x + 3. Then simplify.

$f(x+3) = 2(x+3) - 5$
$f(x+3) = 2x + 6 - 5$
$f(x+3) = 2x + 1$ This is as far as we can go on this one.

c.) $f(x) + f(3)$

For this one we can replace the $f(x)$ with 2x – 5. We also know $f(3)$.

$f(x) + f(3) = 2x - 5 + 1$
$f(x) + f(3) = 2x - 4$ Notice this is not the same as part b, so the f is not distributed to the x and 3.

d.) $f(x+h)$

For this one just replace the x with the expression x + h.

$f(x+h) = 2(x+h) - 5$
$f(x+h) = 2x + 2h - 5$ This is as far as we can go.

Domain: (input) all the x-values that make the equation defined
Defined: There is no division by zero or square roots of negative numbers
Range: (output) all y-values that a graph uses.

EXAMPLE: Find the domain: $y = 2x - 5$

There is no place where you can divide by zero or take the square root of a negative number, so the domain would be all reals, indicated by $(-\infty, \infty)$.

EXAMPLE: Find the domain: $y = \dfrac{1}{2x-5}$

Here it is possible to have a zero in the denominator. The denominator is not allowed to be zero, so solve: $2x - 5 \neq 0$. Solving this you will get $x \neq \dfrac{5}{2}$. This means any number but five halves will work. To write this in interval notation it would be: $\left(-\infty, \dfrac{5}{2}\right) \cup \left(\dfrac{5}{2}, \infty\right)$.

EXAMPLE: Find the domain: $y = \sqrt{2x-5}$

For this one you need to make sure you do not take the square root of a negative number. The only numbers that will work are positive numbers, so solve this equation: $2x - 5 \geq 0$. It is okay for our answer to equal 0. Solving it you will get $x \geq \dfrac{5}{2}$. In interval notation this would look like $\left[\dfrac{5}{2}, \infty\right)$.

EXAMPLE: Find the domain: $y = \dfrac{1}{\sqrt{2x-5}}$

This has two domains restrictions. First the denominator can't be zero. Also we are not allowed to have negative numbers under the square root. We will set it up almost the same as before, but this time we will not include zero. We want to solve: $2x - 5 > 0$. We don't want a zero in the denominator, so we don't include it in our answer. Solving this we get $x > \dfrac{5}{2}$ and the interval notation would be $\left(\dfrac{5}{2}, \infty\right)$.

EXAMPLE: Find the domain: $y = \dfrac{1}{x^2+9}$

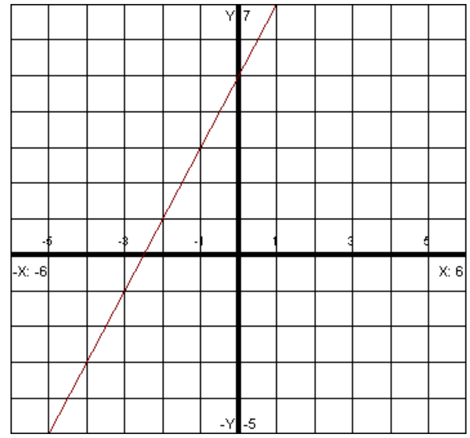
Since we have a fraction we need to set the denominator equal to zero. Let's look at the bottom for a second. Is it possible for us to get a zero on the bottom? The answer is no. If you try 3 as you would suspect is the answer it will not work since it will give you 18 since there is a plus sign. Since the bottom will never be zero that means we have no domain restrictions, so we can use any real number for x. Interval notation: $(-\infty, \infty)$.

EXAMPLE: Find the x and y intercepts and use them to graph the following equation: $6x - 3y + 15 = 0$.

To find an x-intercept, put a zero in for y and solve: $6x - 3(0) + 15 = 0$.

Simplifying will give us $6x + 15 = 0$. Solve for x: $6x = -15$, so $x = -\dfrac{5}{2}$.

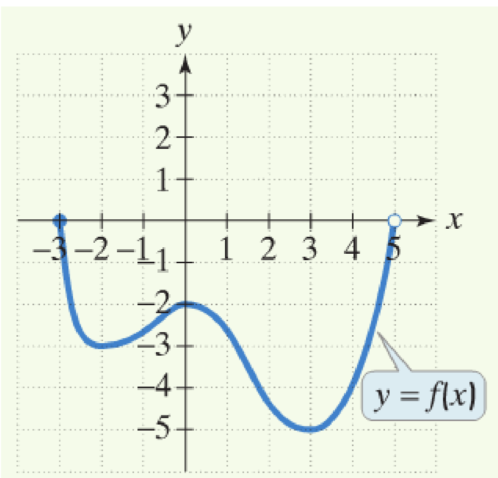
It is important to write our answer as a point. The x-intercept is $\left(-\dfrac{5}{2},\ 0\right)$.



To find an y-intercept, put a zero in for x and solve: $6(0) - 3y + 15 = 0$.
Simplifying will give us $-3y + 15 = 0$. Solve for y: $-3y = -15$, so $y = 5$.
It is important to write our answer as a point. The y-intercept is (0, 5).

To graph, just plot each point. For fractions, you can change them into a decimal. Our x-intercept can be written as: (-2.5, 0).

EXAMPLE: Use the graph below to answer the following questions



a.) Indicate the interval(s) of which f is increasing
$(-2,0) \cup (3,5)$

b.) Indicate the interval(s) of which f is decreasing
$(-3,-2) \cup (0,3)$

c.) List the number(s) where f has a relative minimum.
This is asking for the x value(s) at which the graph has a local minimum. This occurs at x = -2 and at x = 3.

d.) What is the relative maximum(s)?
This is asking for the y-value of the local max, which is -2.

e.) What is the relative minimum(s)?
The y-value of the local minimum is -3 and -5.

f.) What is the domain?
[-3, 5)

g.) What is the range?
[-5, 0]

Composite Functions – a way of combining two functions

$(f \circ g)(x) = f(g(x))$ This is pronounced “f of g of x” DOES NOT MEAN F TIMES G!!!
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These are not multiplications. The $(f \circ g)(x)$ means we place the g function into the f function.
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EXAMPLE: Given: $f(x) = x + 3$ and $g(x) = 2x^2 - 1$ find the following:
$(f \circ g)(x)$, $(g \circ f)(x)$

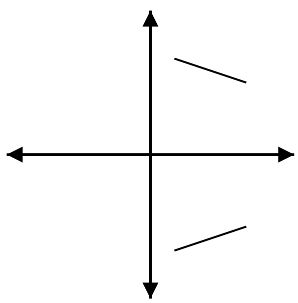
For this one the process is the same as I described about. I will only show the algebraic steps here.

$(f \circ g)(x) = f(g(x))$
$f(2x^2 - 1) = (2x^2 - 1) + 3$, so $(f \circ g)(x) = 2x^2 + 2$

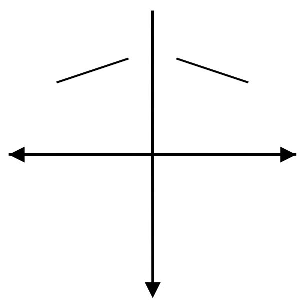
$(g \circ f)(x) = g(f(x))$
$g(x+3) = 2(x+3)^2 - 1 = 2(x^2 + 6x + 9) - 1 = 2x^2 + 12x + 18 - 1$, so $(g \circ f)(x) = 2x^2 + 12x + 17$

Symmetry

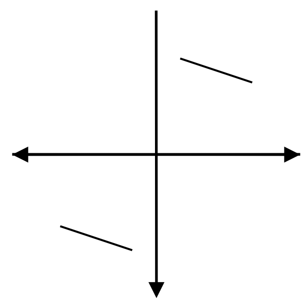
Think if symmetry as a fold line. If a graph can be folded on top of itself and everything overlaps, then it has symmetry. The fold line that allows this to happen is called the line of symmetry. Below is the three types of symmetry that is possible.



x – axis symmetry



y-axis symmetry



origin symmetry

Looking at the above drawings we can come up with relationships. Lets begin with a point (x, y).

Two points symmetric about the x-axis would be (x, y) and (x, -y).
Two points symmetric about the y-axis would be (x, y) and (-x, y).
Two points symmetric about the origin would be (x, y) and (-x, -y).

How to test for symmetry without graphing

x-axis: Replace y with –y in the original equation. If it simplifies to the original equation, it has x-axis symmetry.

y-axis: Replace x with –x in the original equation. If it simplifies to the original equation, it has y-axis symmetry.
.
origin: Replace x with –x and y by -y in the original equation. If it simplifies to the original equation, it has origin symmetry.

EXAMPLE: Test the following equation for symmetry: $y = x^3 - 4x$ and find the intercepts.

x-int: $0 = x^3 - 4x$ Put a zero in for y and solve for x.
$0 = x(x^2 - 4)$
$x = \pm 2$ and x = 0. The points are $(\pm 2, 0)$ and (0, 0).

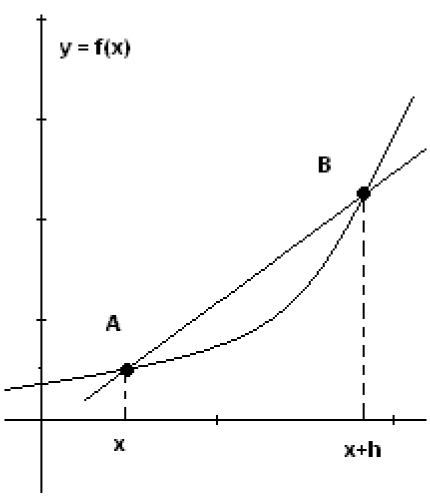
y-int: $y = 0^3 - 4(0)$ Put a zero in for x and solve for y.
$y = 0$ and the point is (0, 0)

x-axis: $-y = x^3 - 4x$ Replace the y with –y in the original equation. This is not the same as what we started with, so it does not have x-axis symmetry.

y-axis: $y = (-x)^3 - 4(-x)$ Replace x with –x and simplify.
$y = -x^3 + 4x$ No matter what this will never be the same as the original, so it does not have this sym.

Origin: $-y = (-x)^3 - 4(-x)$ Replace x with –x and y with –y. Now simplify.
$-y = -x^3 + 4x$ You may think there is no symmetry because it is not the same as the original, but
$-(-y = -x^3 + 4x)$ let's multiply both sides by a negative one so we can solve for y.
$y = x^3 - 4x$ You do get the same as the original so it does have this symmetry.

Difference Quotient



If we wanted to find the slope of a curved line, the only way we can do this is by estimating it with a straight line. We will start with one point and then move over by a small amount h. Now we will use the slope formula. In the picture we have two points, A and B. The coordinates for these are: $(x, f(x))$ and $(x+h, f(x+h))$.

The slope, also called the **difference quotient** is: $\frac{f(x+h) - f(x)}{h}$

In calculus we will try to minimize h so that it is so small that we end up at a point, which will be the exact slope of the curved line at x.

EXAMPLE: Let $f(x) = 3x^2 - x + 1$. Find the difference quotient.

We will do this the same way as above. First we will find $f(x+h)$.

$f(x+h) = 3(x+h)^2 - (x+h) + 1$ What is $(x+h)^2$? If you are thinking $x^2 + h^2$ you are wrong. This is actually $(x+h)(x+h)$ which is a FOIL. It is $x^2 + 2xh + h^2$.

$f(x+h) = 3(x+h)(x+h) - x - h + 1$

$f(x+h) = 3(x^2 + 2xh + h^2) - x - h + 1$

$f(x+h) = 3x^2 + 6xh + 3h^2 - x - h + 1$

Now that we have simplified this as much as possible, we will put it into the difference quotient formula.

$\dfrac{3x^2 + 6xh + 3h^2 - x - h + 1 - (3x^2 - x + 1)}{h}$ Now we will distribute the minus into $f(x)$.

$\dfrac{3x^2 + 6xh + 3h^2 - x - h + 1 - 3x^2 + x - 1}{h}$ Now cancel and simplify.

$\dfrac{6xh + 3h^2 - h}{h}$ Now we can factor out an h from the top.

$\dfrac{h(6x + 3h - 1)}{h}$ Last thing is we can cancel the h from top and bottom.

$6x + 3h - 1$ This is our answer.