1.2 Review of Functions and Graphing

This section does a quick review of functions and graphing.

Slope Formula

The slope formula is used to find the slope between two points (x_1, y_1) and (x_2, y_2):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLE: Find the slope of a line passing through (–1, 3) and (2, 4).

To do this problem we can label our point so we know what to put into the slope formula. It doesn't matter which point you call x_1 or x_2. I will label the point as the following: $x_1 = -1$, $y_1 = 3$, $x_2 = 2$, $y_2 = 4$. Now we plug these into the slope formula: $m = \frac{4-3}{2-(-1)} = \frac{1}{3}$.

Slope-Intercept Formula– this is the standard from of a line which allows you to easily identify the slope and y-intercept.

$y = mx + b$ Here the slope is m and the y-intercept is (0, b).

Point-Slope Formula – this is used when you want to find the equation of a line when you are given a slope and another point on the line. This other point does not need to be the y-intercept.

$y - y_1 = m(x - x_1)$

EXAMPLE: Use the information and given conditions to write an equation for each line in slope-intercept form as well as the point-slope form.

Slope = $-\frac{3}{5}$, passing through (10, -4).

For this one, we know that m = $-\frac{3}{5}$, $x_1 = 10$, and $y_1 = -4$. We can plug these into our point-slope formula: $y - (-4) = -\frac{3}{5}(x - 10)$. When we simplify, we get: $y + 4 = -\frac{3}{5}(x - 10)$. The equation of this line is now written in point-slope form, which is one of our answers. Now we need to write it in slope-intercept form. First we distribute the $-\frac{3}{5}$: $y + 4 = -\frac{3}{5}x - \left(\frac{3}{5}\right)\left(-\frac{10}{1}\right)$. To multiply the two fractions on the end, multiply across the top and bottom. You will get: $y + 4 = -\frac{3}{5}x + 6$. Now subtract 4 from both sides to get our second answer: $y = -\frac{3}{5}x + 2$.

Finding x and y intercepts from an equation

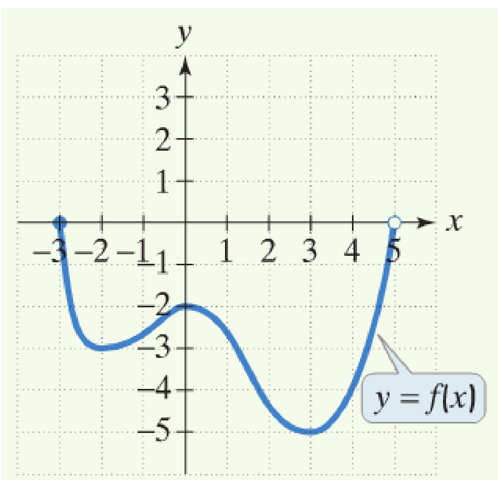
To find the x-intercept, put in a zero for y and solve for x.
To find the y-intercept, put in a zero for x and solve for y.

EXAMPLE: Find the intercepts given $4x + y^2 = 4$.

x-int: $4x + (0)^2 = 4$ Put a zero in for y and solve for x.
$4x = 4$
x = 1, and the point is (1, 0)

y-int: $4(0) + y^2 = 4$ Put a zero in for x and solve for y.
$y^2 = 4$
$y = \pm 2$ and the point is $(0, \pm 2)$

EXAMPLE: Use the graph below to answer the following questions



a.) Indicate the interval(s) of which f is increasing
$(-2,0) \cup (3,5)$

b.) Indicate the interval(s) of which f is decreasing
$(-3,-2) \cup (0,3)$

c.) List the number(s) where f has a relative minimum.
This is asking for the x value(s) at which the graph has a local minimum. This occurs at x = -2 and at x = 3.

d.) What is the relative maximum(s)?
This is asking for the y-value of the local max, which is -2.

e.) What is the relative minimum(s)?
The y-value of the local minimum is -3 and -5.

f.) What is the domain?
[-3, 5)

g.) What is the range?
[-5, 0

Function Definition: For each input (x) there can only be one output (y).

Function notation: $f(x)$ which means "f of x". This does not mean f times x. It means that we have a function called f which contains the variable x.

EXAMPLE: Given the function $f(x)=2x-5$, find the following:

a.) Find $f(3)$. Solve $f(x)=7$.

Whatever is inside the parenthesis goes in place of x in the original expression. This is really asking us for the y value when x is 3.

$$f(3)=2(3)-5$$
$$f(3)=1$$

To solve $f(x)=7$, we put in a 7 for $f(x)$: $7=2x-5$. Now we solve for x. Add the 5 to both sides and divide both sides by 2 to get the answer of 6.

b.) $f(x+3)$

Now we need to replace x in the original equation with x + 3. Then simplify.

$$f(x+3)=2(x+3)-5$$
$$f(x+3)=2x+6-5$$
$f(x+3)=2x+1$ This is as far as we can go on this one.

c.) $f(x+h)$

For this one just replace the x with the expression x + h.

$$f(x+h)=2(x+h)-5$$
$f(x+h)=2x+2h-5$ This is as far as we can go.

d.) $f(-x)$

For this one, replace the x with -x: $f(-x)=2(-x)-5 \quad \Rightarrow \quad f(-x)=-2x-5$

e.) $-f(x)$

This expression means $-1\cdot f(x)$

$$-f(x)=-\left(2x-5\right)$$
$$-f(x)=-2x+5$$

Domain: (input) all the x-values that make the equation defined
Defined: There is no division by zero or square roots of negative numbers
Range: (output) all y-values that a graph uses.

EXAMPLE: Find the domain: $y=2x-5$

There is no place where you can divide by zero or take the square root of a negative number, so the domain would be all reals, indicated by $\left(-\infty,\infty\right)$.

EXAMPLE: Find the domain: $y=\dfrac{x+1}{2x-5}$

Here it is possible to have a zero in the denominator. The denominator is not allowed to be zero, so solve: $2x-5\neq 0$. Solving this you will get $x\neq \dfrac{5}{2}$. This means any number but five halves will work. To write this in interval notation it would be: $\left(-\infty,\dfrac{5}{2}\right)\cup\left(\dfrac{5}{2},\infty\right)$.

EXAMPLE: Find the domain: $y=\sqrt{2x-5}$

For this one you need to make sure you do not take the square root of a negative number. The only numbers that will work are positive numbers, so solve this equation: $2x-5\geq 0$. It is okay for our answer to equal 0. Solving it you will get $x\geq \dfrac{5}{2}$. In interval notation this would look like $\left[\dfrac{5}{2},\infty\right)$.

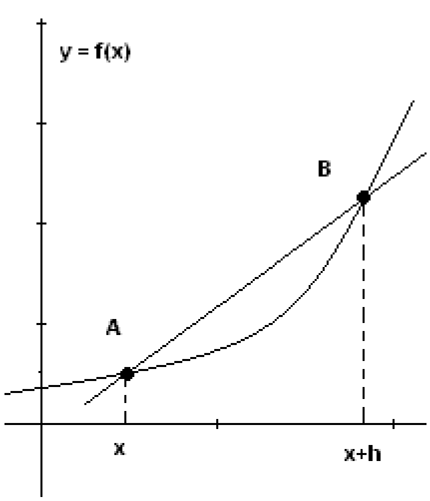
EXAMPLE: Find the domain: $y=\dfrac{3}{\sqrt{2x-5}}$

This has two domains restrictions. First the denominator can't be zero. Also we are not allowed to have negative numbers under the square root. We will set it up almost the same as before, but this time we will not include zero. We want to solve: $2x-5>0$. We don't want a zero in the denominator, so we don't include it in our answer. Solving this we get $x>\dfrac{5}{2}$ and the interval notation would be $\left(\dfrac{5}{2},\infty\right)$.

EXAMPLE: Find the domain: $y=\dfrac{1}{x^2+9}$

Since we have a fraction we need to set the denominator equal to zero. Let's look at the bottom for a second. Is it possible for us to get a zero on the bottom? The answer is no. If you try 3 as you would suspect is the answer it will not work since it will give you 18 since there is a plus sign. Since the bottom will never be zero that means we have no domain restrictions, so we can use any real number for x. Interval notation: $(-\infty,\infty)$.

Difference Quotient



If we wanted to find the slope of a curved line, the only way we can do this is by estimating it with a straight line. We will start with one point and then move over by a small amount h. Now we will use the slope formula. In the picture we have two points, A and B. The coordinates for these are: $(x, f(x))$ and $(x+h, f(x+h))$.

The slope, also called the **difference quotient** is: $\dfrac{f(x+h)-f(x)}{h}$

In calculus we will try to minimize h so that it is so small that we end up at a point, which will be the exact slope of the curved line at x.

EXAMPLE: Let $f(x) = 2x - 3$. Find the difference quotient.

Let's first find $f(x+h)$. Once we have this we can put it into the difference quotient formula. Replace x in the original equation with x + h.

$f(x+h) = 2(x+h) - 3$ Now simplify.
$f(x+h) = 2x + 2h - 3$

We are ready to substitute this into the difference quotient formula. We have $f(x+h)$ and we also know $f(x)$, which is the original equation.

$\frac{2x + 2h - 3 - (2x - 3)}{h}$ Here we have substituted into the formula. Notice the parenthesis around $f(x)$.

$\frac{2x + 2h - 3 - 2x + 3}{h}$ Now we distributed the minus sign and the last thing is to simplify.

$\frac{2h}{h} = 2$ The 2x and the 3 canceled and then the h canceled, leaving us with our answer of 2

EXAMPLE: Let $f(x) = 3x^2 - x + 1$. Find the difference quotient.

We will do this the same way as above. First we will find $f(x+h)$.

$f(x+h) = 3(x+h)^2 - (x+h) + 1$ What is $(x+h)^2$? If you are thinking $x^2 + h^2$ you are wrong. This is actually $(x+h)(x+h)$ which is a FOIL. It is $x^2 + 2xh + h^2$.

$f(x+h) = 3(x+h)(x+h) - x - h + 1$
$f(x+h) = 3(x^2 + 2xh + h^2) - x - h + 1$
$f(x+h) = 3x^2 + 6xh + 3h^2 - x - h + 1$

Now that we have simplified this as much as possible, we will put it into the difference quotient formula.

$\frac{3x^2 + 6xh + 3h^2 - x - h + 1 - (3x^2 - x + 1)}{h}$ Now we will distribute the minus into $f(x)$.

$\frac{3x^2 + 6xh + 3h^2 - x - h + 1 - 3x^2 + x - 1}{h}$ Now cancel and simplify.

$\frac{6xh + 3h^2 - h}{h}$ Now we can factor out an h from the top.

$\frac{h(6x + 3h - 1)}{h}$ Last thing is we can cancel the h from top and bottom.

$6x + 3h - 1$ This is our answer.