1.2 Basic Classes of Functions

Slope Formula

The slope formula is used to find the slope between two points (x_1, y_1) and (x_2, y_2):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Positive slopes will increase as you move from left to right.
- Negative slopes will decrease as you move from left to right.
- A slope of zero is a horizontal line.
- An undefined or infinity slope is a vertical line.

EXAMPLE: Find the slope of a line passing through the following points. Indicate whether the line increases, decreases, is horizontal or vertical.

a.) (-1, 3) and (2, 4)

To do this problem we can label our point so we know what to put into the slope formula. It doesn't matter which point you call x_1 or x_2. I will label the point as the following: $x_1 = -1$, $y_1 = 3$, $x_2 = 2$, $y_2 = 4$. Now we plug these into the slope formula: $m = \frac{4-3}{2-(-1)} = \frac{1}{3}$. Since the slope is positive we know this line increases.

b.) (4, -1) and (3, -1)

First we label the point as the following: $x_1 = 4$, $y_1 = -1$, $x_2 = 3$, $y_2 = -1$. Now we plug these into the slope formula: $m = \frac{-1-(-1)}{3-4} = \frac{0}{-1} = 0$. Since the slope is zero we know this line is horizontal.

c.) (3, -2) and (3, -5)

First we label the point as the following: $x_1 = 3$, $y_1 = -2$, $x_2 = 3$, $y_2 = -5$. Now we plug these into the slope formula: $m = \frac{-5-(-2)}{3-3} = \frac{-3}{0} = \textit{undefined}$. Since the slope is undefined we know this line is vertical.

Slope-Intercept Formula– this is the standard from of a line which allows you to easily identify the slope and y-intercept.

$y = mx + b$ Here the slope is m and the y-intercept is (0, b).

Linear Function– this is the same as the slope-intercept form, except with function notation. In general, a linear function begins with $f(x)$ and contains an x with a power of 0 or 1.

$$f(x) = mx + b$$

Point-Slope Formula – this is used when you want to find the equation of a line when you are given a slope and another point on the line. This other point does not need to be the y-intercept.

$$y - y_1 = m(x - x_1)$$

EXAMPLE: Use the information and given conditions to write an equation for each line in slope-intercept form as well as the point-slope form.

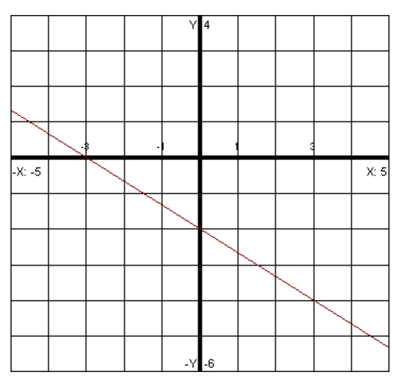
Passing through (-3, 6) and (3, -2)

This time we are not given a slope, so we first must use the slope formula. We label our points and put them into the slope formula: $m = \frac{-2-6}{3-(-3)} = \frac{-8}{6} = -\frac{4}{3}$. When we use the point-slope formula we can use EITHER of our given points as the $(x_1,\ y_1)$. In this case I will use the first point. So $x_1 = -3$, and $y_1 = 6$. We can plug these into our point-slope formula:

$y - 6 = -\frac{4}{3}(x - (-3))$. When we simplify we get: $y - 6 = -\frac{4}{3}(x + 3)$. The equation of this line is now written in point-slope form, which is one of our answers. Now we need to write it in slope-intercept form. First, we distribute the $-\frac{4}{3}$: $y - 6 = -\frac{4}{3}x - 4$. Now add 6 to both sides to get our second answer: $y = -\frac{4}{3}x + 2$.

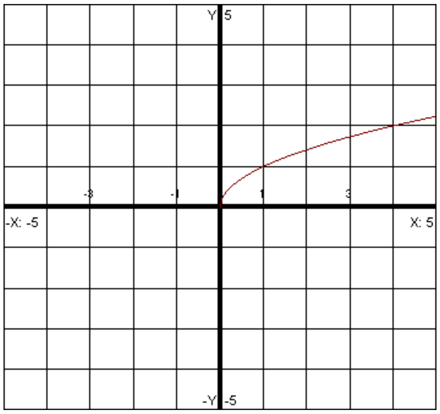
EXAMPLE: Write the following $4x + 6y = -12$ in slope-intercept form and identify the slope and y-intercept. Use this information to graph the equation.

We need to solve for y in order to put this into slope-intercept form. First isolate y: $6y = -4x - 12$. Now divide both sides by 6 to get: $y = -\frac{2}{3}x - 2$. Now we can identify that the slope is $-\frac{2}{3}$ and the y-intercept is



(0, -2). To graph this, first plot the y-intercept. Now the fraction $-\frac{2}{3}$ can be written as either $\frac{-2}{3}$ or $\frac{2}{-3}$. If we think of the slope as $\frac{-2}{3}$ then the our vertical change is -2. This means we move DOWN 2 units from our y-intercept. The bottom number is 3, so we will need to move 3 units to our right. So from our y-int we will move DOWN 2 units and 3 units to the right. This will give us our next point. Plot this and connect our two points with a line. If we used $\frac{2}{-3}$ then we would move UP two units and to the LEFT 3 units. Notice we will still get another point on the same line, so we can use either fraction.

The following is a library of functions that you should know since sketches are sometimes necessary in Calculus.



$y=\sqrt{x}$

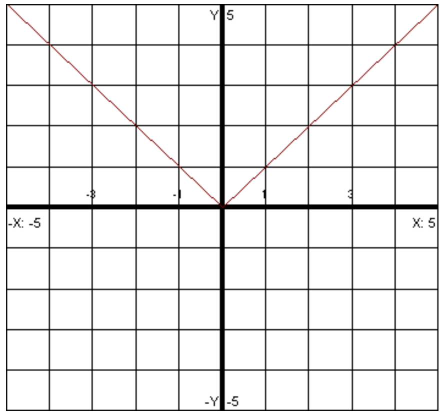
Square Root Function

Domain: $[0,\ \infty)$

Range: $[0,\ \infty)$

Increasing: $(0,\ \infty)$

Decreasing: None



$y=|x|$

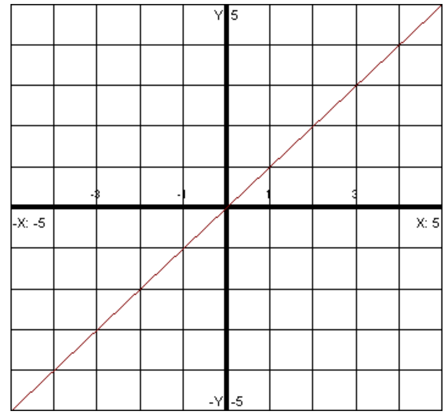
Absolute Value Function

Domain: $(-\infty,\ \infty)$

Range: $[0,\ \infty)$

Increasing: $(0,\ \infty)$

Decreasing: $(-\infty,\ 0)$



$y=x$

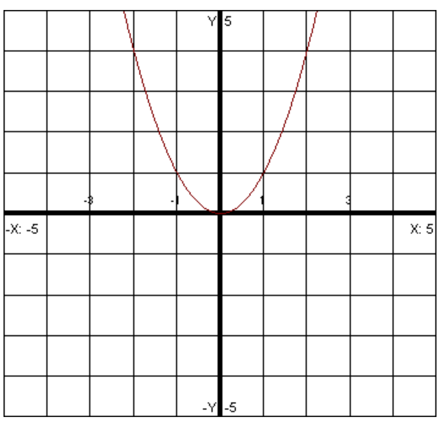
Identity Function

Domain: $(-\infty,\ \infty)$

Range: $(-\infty,\ \infty)$

Increasing: $(-\infty,\ \infty)$

Decreasing: None



$y=x^2$

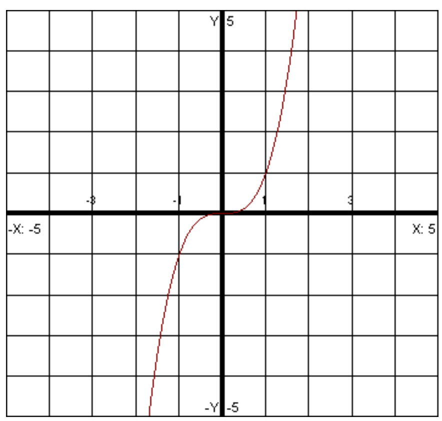
Standard Quadratic Function

Domain: $(-\infty,\ \infty)$

Range: $[0,\ \infty)$

Increasing: $(0,\ \infty)$

Decreasing: $(-\infty,\ 0)$



$y=x^3$

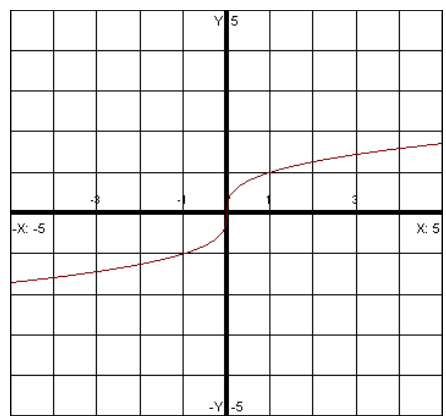
Standard Cube Function

Domain: $(-\infty,\ \infty)$

Range: $(-\infty,\ \infty)$

Increasing: $(-\infty,\ \infty)$

Decreasing: None



$y=\sqrt[3]{x}$

Cube Root Function

Domain: $(-\infty,\ \infty)$

Range: $(-\infty,\ \infty)$

Increasing: $(-\infty,\ \infty)$

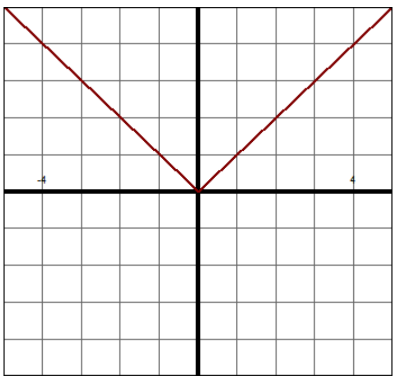
Decreasing: None

Transformations and Graph Sketches

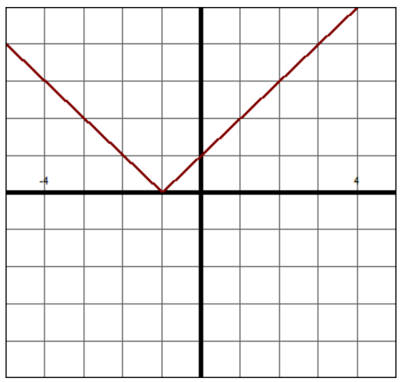
Suppose $y = f(x)$ is the original function (one we looked at in a previous section)

$y = f(x) + k$ moves $f(x)$ k units up
$y = f(x) - k$ moves $f(x)$ k units down
$y = f(x - h)$ moves $f(x)$ h units to the right
$y = f(x + h)$ moves $f(x)$ h units to the left
$y = -f(x)$ flips the graph over the horizontal axis
$y = f(-x)$ flips the graph over the vertical axis
$y = a \cdot f(x)$ If $|a| > 1$ then there is a vertical stretch. If $0 < |a| < 1$, then there is a vertical compression.

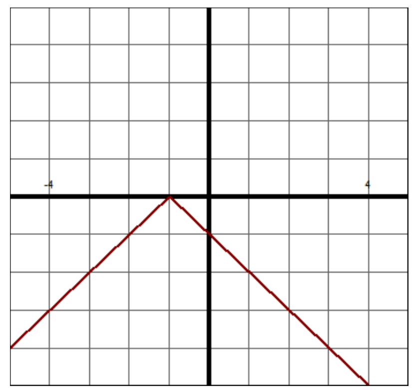
EXAMPLE: Sketch $y = -|x+1| + 2$ by using transformations.



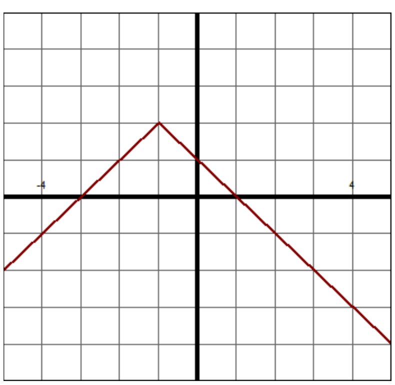
First start with the correct base graph of $y = |x|$.



To graph $y = |x+1|$ we will move the graph $y = |x|$ one unit to the left.



Next we will graph $y = -|x+1|$ To do this we will flip the graph $y = |x+1|$ over the vertical axis.



Finally we will move $y = -|x+1|$ up 2 units. We now have the graph $y = -|x+1| + 2$, which is our answer.

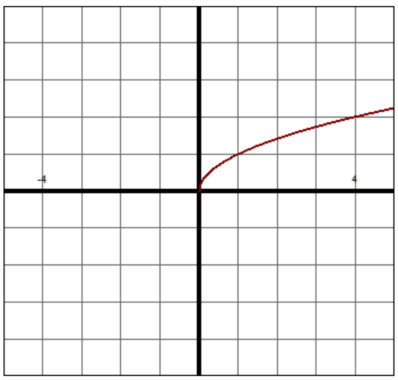
EXAMPLE: Sketch $y = \sqrt{4-x} + 2$ by using transformations.

In order to use the transformation rules the x must come first and there must be a one in front of x. In our problem above we need to first put the x first and then we will factor out a negative:

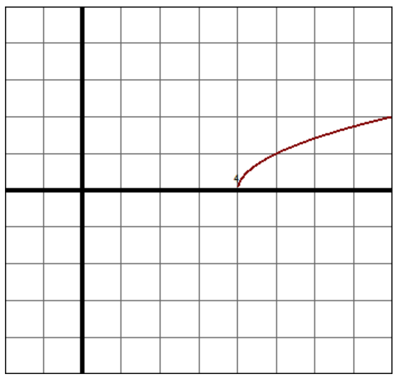
$y = \sqrt{4-x} + 2$

$y = \sqrt{-x+4} + 2$ Here we put the x term first

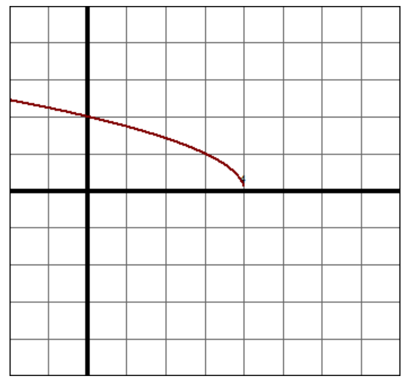
$y = \sqrt{-(x-4)} + 2$ Here we factored out a -1. Now we will graph it.



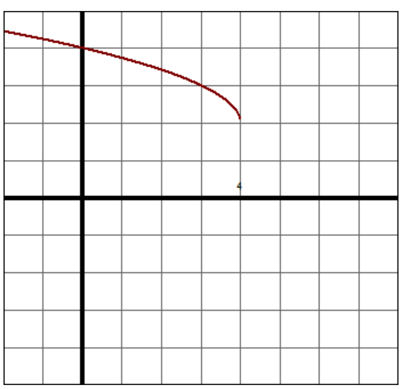
First start with the correct base graph of $y = \sqrt{x}$.



To graph $y = \sqrt{x-4}$ we will move the graph $y = \sqrt{x}$ four units to the right.



To graph $y = \sqrt{-(x-4)}$ we will flip $y = \sqrt{x-4}$ over the vertical axis.



Now we will move the graph $y = \sqrt{-(x-4)}$ up two units.
This graph is our final answer, which is $y = \sqrt{4-x} + 2$.

Piecewise Functions

These functions are made up of different pieces. Each piece is defined for certain values of x.

EXAMPLE: Use the function $f(x)=\begin{cases} x+2 & if \quad x<-3 \\ x-2 & if \quad x\geq -3 \end{cases}$ to find $f(-4)$, $f(-3)$ and $f\left(-\frac{3}{2}\right)$. Then graph. and use this to determine the graph's range.

a.) $f(-4)$ In order to know which equation we are using, look at the number inside the parenthesis, which is -4. In our function, we need to find what function includes -4. This would be the first equation since -4 is less than -3. So we put -4 in for x in the first equation. You will get -4 + 2 = -2. So $f(-4)=-2$.

b.) $f(-3)$ The equation that includes -3 would be the second one since it is greater than or equal to -3. So we will place the -3 in for x in the second equation: -3 – 2 = -5. So $f(-3)=-5$.

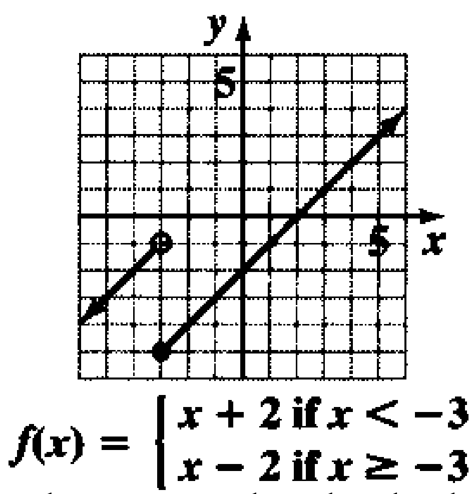
c.) $f\left(-\frac{3}{2}\right)$ If you don't know if this fraction is less or more than -3 then turn it into a decimal. This is -1.5.

So this would be greater than -3, so we will use the second equation. $-\frac{3}{2}-2=\frac{-7}{2}$. So $f\left(-\frac{3}{2}\right)=-\frac{7}{2}$.

Now we do a graph. Notice that there are two different equations we need to graph. To graph these, we will make a table of values for each one. The most important thing is that we need to plug in x values that match our conditions in the problem. For example the first equation says we need to use x values that are less than but not equal to -3. Now we can plug in -3, and this will end up as an open circle on the graph since it is not included. So we can use x = -5, -4, -3. Three points are enough. For the second equation we need to pick x values that are greater than or equal to -3. When we plug in -3 we can plot this point as a closed circle since this point is included. We will use x = -3, -2, -1. Below are a table of values for each equation. To graph this, we plot points from our table making sure to indicate a closed or open circle:

x	$y=x+2$	(x, y)
-5	$y=-5+2=-3$	(-5, -3)
-4	$y=-4+2=-2$	(-4, -2)
-3	$y=-3+2=-1$	(-3, -1)

x	$y=x-2$	(x, y)
-3	$y=-3-2=-5$	(-3, -5)
-2	$y=-2-2=-4$	(-2, -4)
-1	$y=-1-2=-3$	(-1, -3)



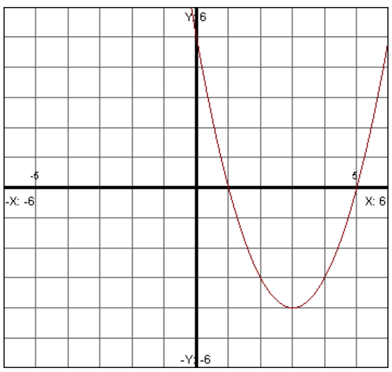
Notice the open circle at (-3, -1). Also notice that there are no x values plotted to the right of the open circle because this graph is only defined for values less than or equal to -3. Notice the closed circle at (-3, -5). Notice that there are not x values plotted to the left of this point since we are only allowed to use values for x that are greater than or equal to -3. The range is the y-values the graph is using. This would be ALL y values, so the answer is $(-\infty,\ \infty)$.

EXAMPLE: Find the intercepts, vertex, axis of symmetry, domain, range, and the graph of $y = x^2 - 6x + 5$.

First we will find the x-intercept. Put in a zero for y. You will get $0 = x^2 - 6x + 5$. In order to solve this you must factor. You will get $0 = (x-1)(x-5)$. Solving you will get x = 1 and x = 5, or (1, 0) and (5, 0) in intercept form. To find the y-intercept, put in a 0 for x. You will get y = 5, or (0, 5) in intercept form. Now we need to find the vertex. We will use the vertex formula so find the x coordinate of the vertex:

$x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$. So now we will put in a 3 for x in the original equation to get the y-value of the vertex.

$y = (3)^2 - 6(3) + 5$ so y = -4. The vertex is (3, -4). Now we just need to plot the points to get the graph.



Notice that the number in front of the x^2 is greater than zero, so the parabola opens up and the vertex is the lowest point on the graph.

Here the domain (x values) is: $(-\infty,\ \infty)$.
The range (y values) is $[-4,\ \infty)$

EXAMPLE: Suppose the height of an object shot straight up is given by $h = 512t - 16t^2$ where h is measured in feet and t is in seconds. Find the maximum height and the time at which the object hits the ground.

This problem is easier that the previous ones since we don't need to first figure out the formula. You want to find the vertex. This will give us the time at which the maximum height will occur. Use the vertex formula:

$t = \frac{-512}{2(-16)} = 16$. So we know it takes 16 seconds for the object to reach its maximum height. To find the height, put in a 16 for each t in $h = 512t - 16t^2$. You will get: $h = 512(16) - 16(16)^2 = 4096 ft$.

The second part of the problem asks us to find the time at which the object hits the ground. This occurs when the height is zero. So put in a zero for h and solve for t: $0 = 512t - 16t^2$. We will solve by factoring.

$0 = 512t - 16t^2$
$0 = 16t(32 - t)$ Solving this you will get. $t = 32$ seconds. You also get t = 0 as a solution.