NAME: KEY

MATH 126 FINAL EXAM SAMPLE KEY

NOTE: The final exam will only have 14 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A.) (5 pts) Find the intercepts and indicate what kind of symmetry this graph has: $y^2 = x + 4$.

<u>Intercepts:</u>

x-int: $y = 0$

$0^2 = x + 4$

$x = -4$

y-int: $x = 0$

$y^2 = 0 + 4$

$y^2 = 4$

$y = \pm 2$

<u>Symmetry:</u>

x-axis: -y for y

$(-y)^2 = x + 4$

$y^2 = x + 4$

Same as original so it has this symmetry.

y-axis: -x for x

$y^2 = (-x) + 4$

$y^2 = -x + 4$

Not same as original so it does not have this symmetry.

origin: -x for x and -y for y

$(-y)^2 = (-x) + 4$

$y^2 = -x + 4$

Not same as original so it does not have this symmetry.

x-int: -4

y-int: ± 2

Symmetry: x-axis

1B.) (5 pts) Find the intercepts and indicate what kind of symmetry this graph has: $x^3 + y^3 = 9x$.

<u>Intercepts:</u>

x-int: $y = 0$

$x^3 + 0^3 = 9x$

$x^3 = 9x$

$x^3 - 9x = 0$

$x(x^2 - 9) = 0$

$x = 0, \pm 3$

y-int: $x = 0$

$0^3 + y^3 = 9(0)$

$y^3 = 0$

$y = 0$

<u>Symmetry:</u>

x-axis: -y for y

$x^3 + (-y)^3 = 9x$

$x^3 - y^3 = 9x$

Not same as original so it does not have this symmetry.

y-axis: -x for x

$(-x)^3 + y^3 = 9(-x)$

$-x^3 + y^3 = -9x$

Not same as original so it does not have this symmetry.

origin: -x for x and -y for y

$(-x)^3 + (-y)^3 = 9(-x)$

$-x^3 - y^3 = -9x$ Multiply both sides by -1.

$x^3 + y^3 = 9x$

Same as original so it does have this symmetry.

x-int: $0, \pm 3$

y-int: 0

Symmetry: origin

2A.) (3 pts) Find the domain: $y=\frac{2x-7}{x^3+16x}$. Write in interval notation. 2A. $(-\infty,0)\cup(0,\infty)$

$$x^3+16x=0$$
$$x(x^2+16)=0$$
$$x=0 \quad \text{or} \quad x^2+16\neq 0$$
$$x^2=-16$$

We know $x^2=-16$ will not result in a real number, so we know the factor x^2+16 cannot be 0. Therefore the only way the denominator will be zero is when x = 0. Therefore our domain is all number not including 0. We write this in interval notation as $(-\infty,0)\cup(0,\infty)$.

2B.) (3 pts) Find the domain: $y=\frac{8x-3}{\sqrt{11-5x}}$. Write in interval notation. 2B. $\left(-\infty,\frac{11}{5}\right)$

$$11-5x>0$$
$$-5x>-11$$
$$x<\frac{11}{5}$$

$\left(-\infty,\frac{11}{5}\right)$ We do not include $\frac{11}{5}$ since it makes the bottom zero.

2C.) (3 pts) Find the domain: $y=\frac{\sqrt{3x-4}}{45}$. Write in interval notation. 2C. $\left[\frac{4}{3},\infty\right)$

$$3x-4\geq 0$$
$$3x\geq 4$$
$$x\geq\frac{4}{3}$$

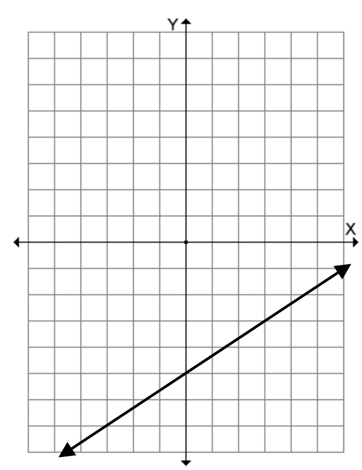
$\left[\frac{4}{3},\infty\right)$ We do include $\frac{4}{3}$ since it does not make the bottom zero.

3A.) (4 pts) Find the equation of a line (in slope-intercept form) that passes through $\left(\frac{3}{2},-4\right)$ and (-3, -7).

Graph your equation.

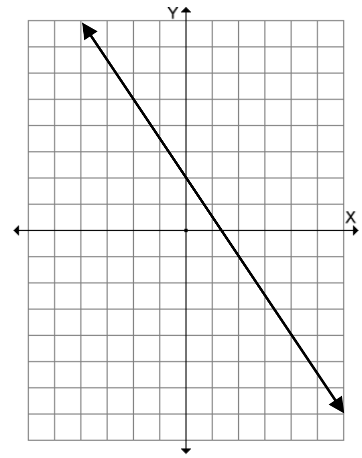
$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-7-(-4)}{-3-\frac{3}{2}}=\frac{-3}{-\frac{9}{2}}=3\cdot\frac{2}{9}=\frac{2}{3}$$

$$y-y_1=m(x-x_1)$$
$$y-(-7)=\frac{2}{3}(x-(-3))$$
$$y+7=\frac{2}{3}(x+3)$$
$$y+7=\frac{2}{3}x+2$$
$$y=\frac{2}{3}x-5$$



Equation: $y=\frac{2}{3}x-5$

3B.) (4 pts) Find the equation of a line (in slope-intercept form) that is perpendicular to $2x - 3y = 4$ and passes through (-2, 5). Graph your equation.



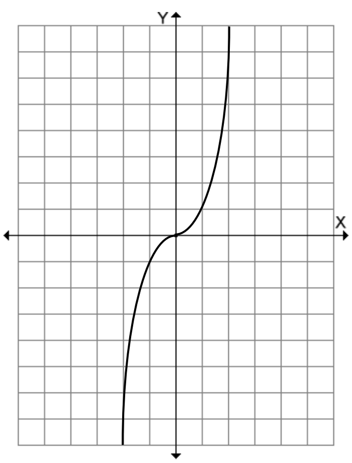
$$2x - 3y = 4$$
$$-3y = -2x + 4$$
$$y = \frac{2}{3}x - \frac{4}{3}$$

Slope perpendicular is $-\frac{3}{2}$

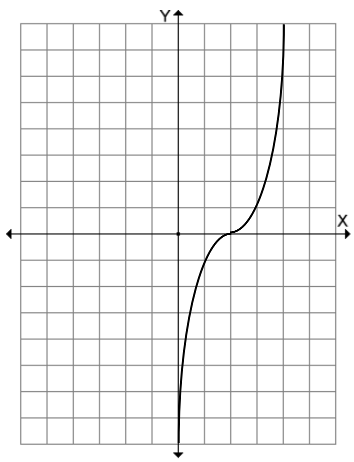
$$y - y_1 = m(x - x_1)$$
$$y - 5 = -\frac{3}{2}(x - (-2))$$
$$y - 5 = -\frac{3}{2}(x + 2)$$
$$y - 5 = -\frac{3}{2}x - 3$$
$$y = -\frac{3}{2}x + 2$$

Equation: $y = -\frac{3}{2}x + 2$

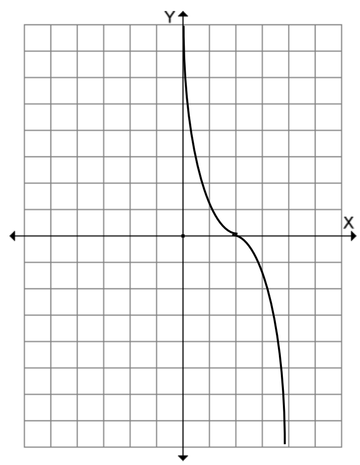
4A.) (5 pts) Graph using transformations: $y = -(x-2)^3 - 1$. Start with the base graph (library function) and then graph each successive transformation. The final graph will be your graph of $y = -(x-2)^3 - 1$.



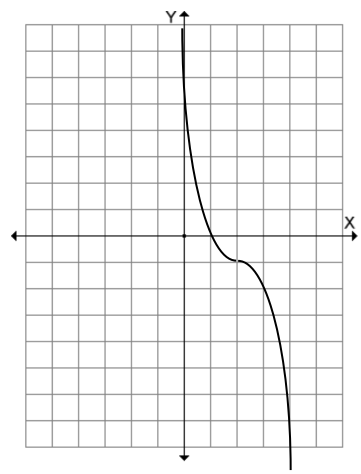
$y = x^3$



$y = (x-2)^3$

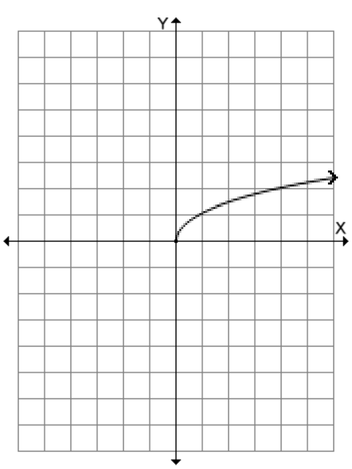


$y = -(x-2)^3$

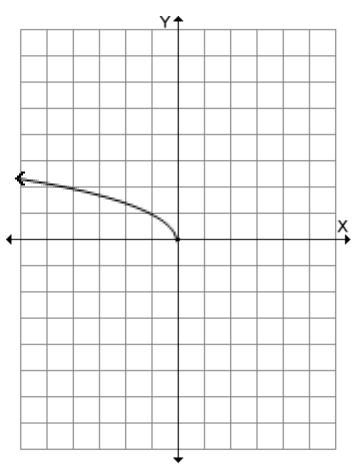


$y = -(x-2)^3 - 1$

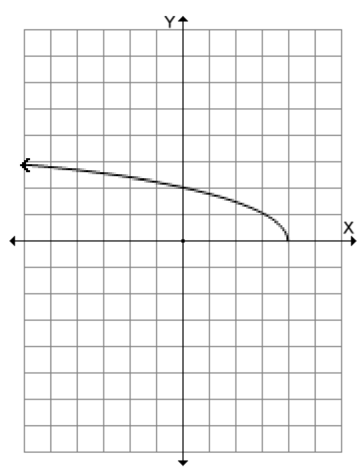
4B.) (5 pts) Graph using transformations: $y = \sqrt{-(x-4)} - 1$. Start with the base graph (library function) and then graph each successive transformation. The final graph will be your graph of $y = \sqrt{-(x-4)} - 1$.



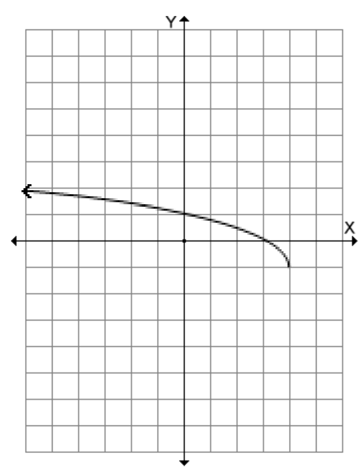
$y = \sqrt{x}$



$y = \sqrt{-x}$



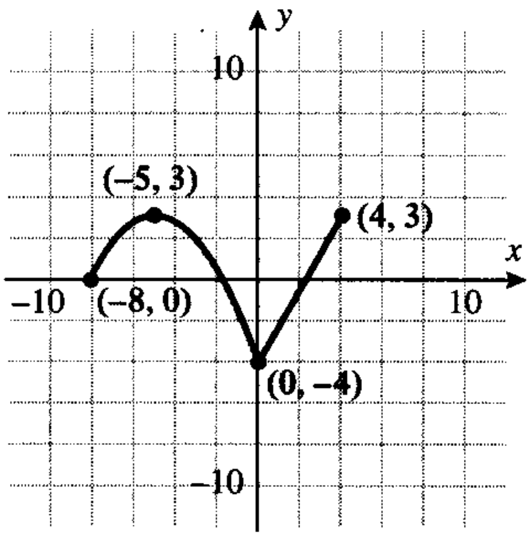
$y = \sqrt{-(x-4)}$



$y = \sqrt{-(x-4)} - 1$

NOTE: For question 5 not all these parts will be given, but you need to know how to do all of them since any of them could be on the exam.

5.) (6 pts.) Use the graph of f(x) below to find the following.



a.) domain: $[-8, 4]$

b.) range: $[-4, 3]$

c) number(s) for which f has a local max.

-5

d) number(s) for which f has a local min.

0

e.) minimum value: -4

f.) maximum value: 3

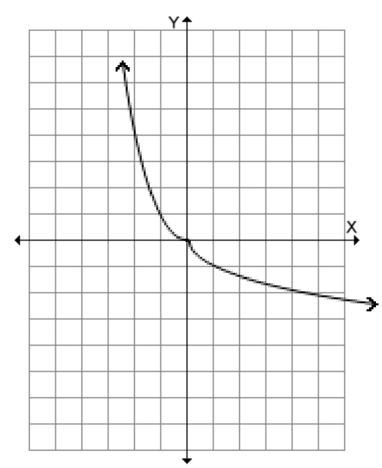
g.) Find $f(-8)$: 0

h.) When does $f(x) = 3$? -5 , 4

i.) Interval(s) at which f is increasing
$(-8,-5) \cup (0,4)$

j.) Interval(s) at which f is decreasing
$(-5,0)$

6A.) (5 pts) Given $f(x) = \begin{cases} x^2 & \text{if} \quad x \le 0 \\ -\sqrt{x} & \text{if} \quad x > 0 \end{cases}$ find the following and graph.



i.) $f(0) = 0^2 = 0$

ii.) $f(9) = -\sqrt{9} = -3$

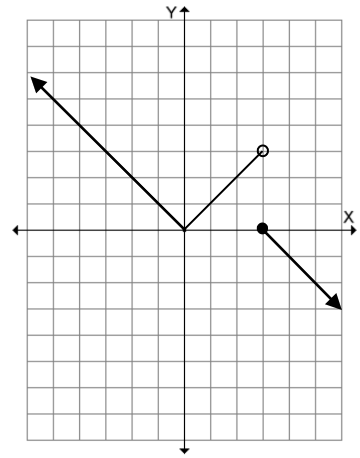
iii.) $f\left(\frac{9}{4}\right) = -\sqrt{\frac{9}{4}} = -\frac{3}{2}$

iv.) $f(-4) = (-4)^2 = 16$

Graph looks like x^2 when x is less than or equal to 0.
Graph looks like $-\sqrt{x}$ when x is greater than 0.

6B.) (5 pts) Given $f(x)=\begin{cases} |x| & if \quad x<3 \\ -x+3 & if \quad x\geq 3 \end{cases}$ find the following and graph.

i.) $f(0)=|0|=0$



ii.) $f(3)=-3+3=0$

iii.) $f(-3)=|-3|=3$

Graph looks like $|x|$ when x is less than 3.

iv.) $f\left(\frac{7}{2}\right)=-\frac{7}{2}+3=-\frac{1}{2}$

Graph looks like $-x+3$ when x is greater than or equal to 3.

7A.) (4 pts) Given $f(x)=2x+1$ and $g(x)=\frac{1}{x^2-4}$, find

i.) $(f\circ g)(x)$ Write as a single fraction.

i. $\frac{x^2-2}{(x+2)(x-2)}$

$$(f\circ g)(x)=f(g(x))=f\left(\frac{1}{x^2-4}\right)$$
$$=2\left(\frac{1}{x^2-4}\right)+1$$
$$=\frac{2}{x^2-4}+1\left(\frac{x^2-4}{x^2-4}\right)$$
$$=\frac{2+x^2-4}{x^2-4}=\frac{x^2-2}{(x+2)(x-2)}$$

ii.) $(g\circ f)(x)$ Factor if possible.

ii. $\frac{1}{(2x-1)(2x+3)}$

$$(g\circ f)(x)=g(f(x))=g(2x+1)$$
$$=\frac{1}{(2x+1)^2-4}$$
$$=\frac{1}{4x^2+4x+1-4}$$
$$=\frac{1}{4x^2+4x-3}=\frac{1}{(2x-1)(2x+3)}$$

7B.) (4 pts) Given $f(x)=\frac{1}{\sqrt{x}}$ and $g(x)=x^2-6x+9$, find :

i.) $(f\circ g)(x)$ (Fully factor and simplify your answer.) i. $\frac{1}{x-3}$

$$(f\circ g)(x)=f(g(x))=f\left(x^2-6x+9\right)$$
$$=\frac{1}{\sqrt{x^2-6x+9}}$$
$$=\frac{1}{\sqrt{(x-3)^2}}=\frac{1}{x-3}$$

ii.) $(g\circ f)(1)$ ii. 4

$$(g\circ f)(1)=g(f(1))=g\left(\frac{1}{\sqrt{1}}\right)=g(1)=1^2-6(1)+9=4$$

8A.) (4 pts) Let $f(x)=3-2x^2$. Find the difference quotient. 8A. $-4x-2h$

Use $\frac{f(x+h)-f(x)}{h}$.

$$f(x+h)=3-2(x+h)^2$$
$$f(x+h)=3-2\left(x^2+2xh+h^2\right)$$
$$f(x+h)=3-2x^2-4xh-2h^2$$

$$\frac{f(x+h)-f(x)}{h}=\frac{3-2x^2-4xh-2h^2-(3-2x^2)}{h}$$
$$=\frac{3-2x^2-4xh-2h^2-3+2x^2}{h}=\frac{-4xh-2h^2}{h}=-4x-2h$$

8B.) (4 pts) Let $f(x)=3x^2-\frac{x}{5}$. Find the difference quotient. 8B. $6x+3h-\frac{1}{5}$

Use $\frac{f(x+h)-f(x)}{h}$.

$$f(x+h)=3(x+h)^2-\frac{x+h}{5}$$
$$f(x+h)=3\left(x^2+2xh+h^2\right)-\frac{x+h}{5}$$
$$f(x+h)=3x^2+6xh+3h^2-\frac{x}{5}-\frac{h}{5}$$

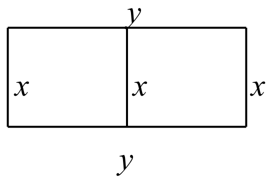
$$\frac{f(x+h)-f(x)}{h}=\frac{3x^2+6xh+3h^2-\frac{x}{5}-\frac{h}{5}-\left(3x^2-\frac{x}{5}\right)}{h}$$

$$=\frac{3x^2+6xh+3h^2-\frac{x}{5}-\frac{h}{5}-3x^2+\frac{x}{5}}{h}=\frac{6xh+3h^2-\frac{h}{5}}{h}=6x+3h-\frac{1}{5}$$

9A.) (4 pts) A rancher as 6000 linear feet of fencing and wants to enclose a rectangular field and then divide it into two equal pastures as shown below. What dimensions produce the maximum enclosed area? What is the maximum area?

Dimensions: 1000 by 1500ft

Max. Area: 1500000 sq. ft.



$$3x+2y=6000$$

$$2y=6000-3x$$

$$y=3000-\frac{3}{2}x$$

$$A=xy$$

$$A=x\left(3000-\frac{3}{2}x\right)$$

$$A=3000x-\frac{3}{2}x^2$$

$$x=\frac{-b}{2a}=\frac{-3000}{2(-3/2)}=\frac{3000}{3}=1000\ \ ft$$

$$y=3000-\frac{3}{2}x$$

$$y=3000-\frac{3}{2}(1000)=3000-1500=1500\ \ ft$$

$$A=xy=1000(1500)=1500000\ \ ft^2$$

9B.) (4 pts) Given $C(x)=680+4x-0.01x^2$ and $R(x)=12x-0.02x^2$, where *x* represents the number of units produced.

i.) Find the amount of units that must be sold to maximize the profit. (**Profit = Revenue – Cost**)

i. 400 units

$$P(x)=R(x)-C(x)$$
$$P(x)=12x-0.02x^2-\left(680+4x-0.01x^2\right)$$
$$P(x)=12x-0.02x^2-680-4x+0.01x^2$$
$$P(x)=12x-0.02x^2-680-4x+0.01x^2$$
$$P(x)=-0.01x^2+8x-680$$

$$x=\frac{-b}{2a}=\frac{-8}{2(-0.01)}=400\ \text{units}$$

ii.) What is the maximum profit?

ii. $920

$$P(x)=-0.01x^2+8x-680$$
$$P(400)=-0.01(400)^2+8(400)-680$$
$$P(400)=\$920$$

9C.) (4 pts) A model rocket is launched. The height, in feet, of the rocket h(t) at t seconds after launch is determined by the equation $h(t) = -\frac{1}{2}t^2 + 15t$.

i.) Find the number of seconds after launch it takes for the rocket to reach its maximum height.

i. 15 seconds

$$t = \frac{-b}{2a} = \frac{-15}{2\left(-\frac{1}{2}\right)} = 15 \text{ seconds}$$

ii.) Find the maximum height obtained by the rocket.

ii. Either $\frac{225}{2}$ *or* 112.5 ft

$$h(15) = -\frac{1}{2}(15)^2 + 15(15)$$

$$h(15) = -\frac{1}{2}(15)^2 + 15(15)$$

$$h(15) = \frac{225}{2} \ \ or\ \ 112.5 \text{ feet}$$

10A.) (4 pts) The function $f(x) = 6x^3 - 5x^2 - 29x + 10$ has a zero at $x = -2$. Use synthetic division and factoring to find the other zeros.

10A. $x = \frac{5}{2}, \frac{1}{3}$

-2 |	6	-5	-29	10
		-12	34	-10
	6	-17	5	0

$$6x^2 - 17x + 5 = 0$$
$$(2x-5)(3x-1) = 0$$
$$x = \frac{5}{2}, \frac{1}{3}$$

10B.) (4 pts) The function $f(x) = 27x^3 - 54x^2 + 27x - 4$ has a zero at $x = \frac{4}{3}$. Use synthetic division and factoring to find the other zeros.

10B. $x = \frac{1}{3}$

4/3 |	27	-54	27	-4
		36	-24	4
	27	-18	3	0

$$27x^2 - 18x + 3 = 0$$
$$3(9x^2 - 6x + 1) = 0$$
$$3(3x-1)(3x-1) = 0$$
$$x = \frac{1}{3}$$

11A.) (4 pts) Solve and write in interval notation: 11A. $\left(-\infty,\frac{1}{2}\right)\cup\left(\frac{7}{3},4\right)$

$(2x-1)(3x-7)(x-4)<0$

	-	+	-	+
$2x-1$	-	+	+	+
$3x-7$	-	-	+	+
$x-4$	-	-	-	+

1/2 7/3 4

test 0 test 1 test 3 test 5

11B.) (4 pts) Solve and write in interval notation: 11B. $(-2,1]$

$$\frac{(1-x)(x-3)^2}{x+2}\geq 0$$

	-	+	-	-
$1-x$	+	+	-	-
$(x-3)^2$	+	+	+	+
$x+2$	-	+	+	+

-2 1 3

test -3 test 0 test 2 test 4

12A.) (4 pts) Fully expand and simplify: 12A. $3\log_3 x+2\log_3(x-2)-\left(1+\frac{1}{2}\log_3(x^2+5)\right)$

$$\log_3\left[\frac{x^3(x-2)^2}{3\sqrt{x^2+5}}\right]$$

$$\log_3 x^3(x-2)^2-\log_3 3(x^2+5)^{\frac{1}{2}}$$

$$\log_3 x^3+\log_3(x-2)^2-\left(\log_3 3+\log_3(x^2+5)^{\frac{1}{2}}\right)$$

$$3\log_3 x+2\log_3(x-2)-\left(1+\frac{1}{2}\log_3(x^2+5)\right)$$

12B.) (4 pts) Fully expand and simplify: 12B. $1+\frac{1}{2}\ln(y-3)-(\ln x+5\ln z)$

$$\ln\left(\frac{e\sqrt{y-3}}{x\cdot z^5}\right)$$

$$\ln e(y-3)^{\frac{1}{2}}-\ln x\cdot z^5$$

$$\ln e+\ln(y-3)^{\frac{1}{2}}-\left(\ln x+\ln z^5\right)$$

$$1+\frac{1}{2}\ln(y-3)-(\ln x+5\ln z)$$

13A.) (4 pts) Solve for x: $\log_3(x-5)-\log_3(2x+3)=0$ 13A. No Solution

$$\log_3\left(\frac{x-5}{2x+3}\right)=0$$

$$3^0=\frac{x-5}{2x+3}$$

$$1=\frac{x-5}{2x+3}$$

$$2x+3=x-5$$

$x=-8$ Causes a negative number inside the log, so no solution.

13B.) (4 pts) Solve for x: $\log_2(x^2-7x)=3+\log_2(x+2)$ 13B. $x=-1,\ 16$

$$\log_2(x^2-7x)-\log_2(x+2)=3$$

$$\log_2\left(\frac{x^2-7x}{x+2}\right)=3$$

$$2^3=\frac{x^2-7x}{x+2}$$

$$8=\frac{x^2-7x}{x+2}$$

$$x^2-7x=8x+16$$

$$0=x^2-15x-16$$

$$0=(x+1)(x-16)$$

$$x=-1,\ 16$$

14A.) (4 pts) A colony of bacteria grows according to the law of uninhibited growth according to the function $N(t)=100e^{0.045t}$ where N is the number of bacteria and t is in days. How long will it take the population to reach 140 bacteria? 14A. 7.48 days

$$140=100e^{0.045t}$$

$$1.4=e^{0.045t}$$

$$\ln 1.4=\ln e^{0.045t}$$

$$\ln 1.4=0.045t$$

$$t=\frac{\ln 1.4}{0.045}\approx 7.48\ days$$

14B.) (4 pts) The following decay function estimates amount of grams of Bismuth-210 after t days: $A(t)=100e^{-0.1382t}$. How long will it take for the original amount of Bismuth-210 to decay to 38 grams? (Round to the nearest day.) 14B. 7 days

$$38=100e^{-0.1382t}$$

$$0.38=e^{-0.1382t}$$

$$\ln 0.38=\ln e^{-0.1382t}$$

$$\ln 0.38=-0.1382t$$

$$t=\frac{\ln 0.38}{-0.1382}\approx 7\ days$$