NAME: KEY

MATH 126 TEST 3 SAMPLE KEY

NOTE: The actual exam will only have 13 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A.) (4 pts) Find the asymptotes. DO NOT GRAPH:

$$y=\frac{6x^2-17x+5}{2x-3}$$

Vertical: $x=\frac{3}{2}$

Slant: $y=3x-4$

Vertical Asymptote:

bottom = 0

$2x - 3 = 0$

$2x = 3$

$x = 3/2$

Slant:

$$\begin{array}{r} 3x-4 \\ 2x-3\,\overline{)\,6x^2-17x+5} \\ \underline{6x^2-9x} \\ -8x+5 \\ \underline{-8x+12} \\ -7 \end{array}$$

1B.) (4 pts) Find the asymptotes. DO NOT GRAPH:

$$y=\frac{4x^2-7}{2x-1}$$

Vertical: $x=\frac{1}{2}$

Slant: $y=2x+1$

Vertical Asymptote:

bottom = 0

$2x - 1 = 0$

$2x = 1$

$x = 1/2$

Slant:

$$\begin{array}{r} 2x+1 \\ 2x-1\,\overline{)\,4x^2+0x-7} \\ \underline{4x^2-2x} \\ 2x-7 \\ \underline{2x-1} \\ -6 \end{array}$$

2A.) (6 pts) Use the following equation to answer the below questions: $y=\frac{x-2}{x^2-2x-3}$.

i.) Find the intercepts.

x-int: 2

y-int: 2/3

x-int: top=0

$x - 2 = 0$

$x = 2$

y-int: $x = 0$

$y = 2/3$

ii.) Find the asymptotes.

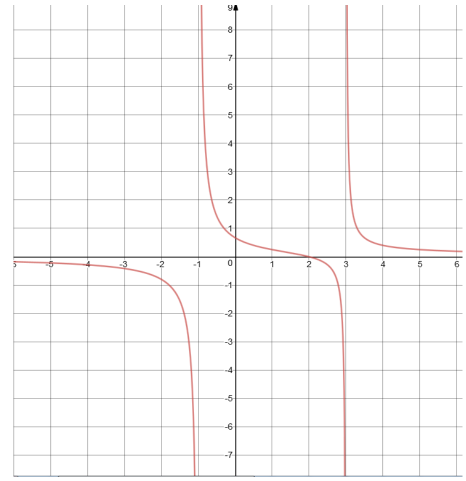
Vertical: $x = -1,\ x = 3$

Vertical: bottom = 0
$x^2 - 2x - 3 = 0$
$(x+1)(x-3) = 0$
$x = -1,\ x = 3$

Horizontal: $n < m$ Rule
$y = 0$

Horizontal: $y = 0$

iii. Graph.



2B.) (6 pts) Use the following equation to answer the below questions: $y = \dfrac{x(x+4)}{x^2 + 4x + 3}$.

i.) Find the intercepts.

x-int: $x = 0,\ x = -4$

x-int: top=0
$x(x+4) = 0$
$x = 0,\ x = -4$

y-int: $x = 0$
y = 0

y-int: 0

ii.) Find the asymptotes.

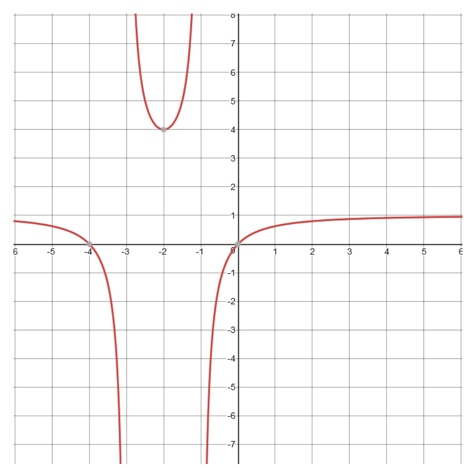
Vertical: $x = -1,\ x = -3$

Vertical: bottom = 0
$x^2 + 4x + 3 = 0$
$(x+1)(x+3) = 0$
$x = -1,\ x = -3$

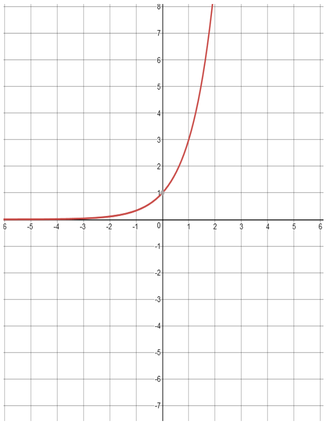
Horizontal: $n = m$ Rule
$y = 1$

Horizontal: $y = 1$

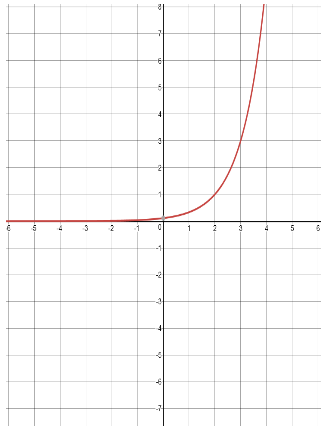
iii. Graph.



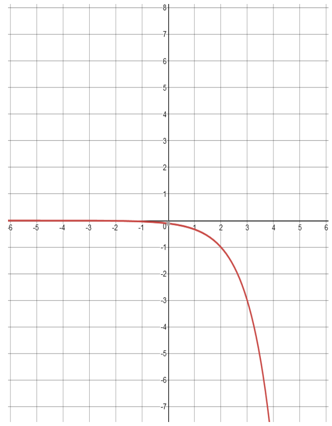
3A.) (5 pts) Graph using transformations: $y=-3^{x-2}+3$. Start with the base graph $y=3^x$ and then graph each successive transformation. The final graph will be your graph of $y=-3^{x-2}+3$.



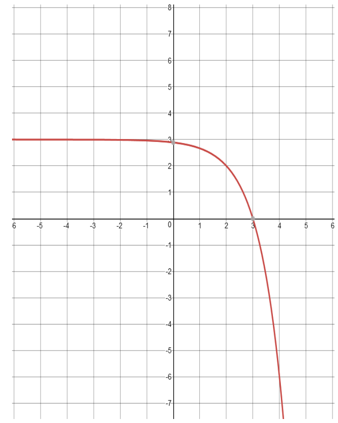
$y=3^x$



$y=3^{x-2}$



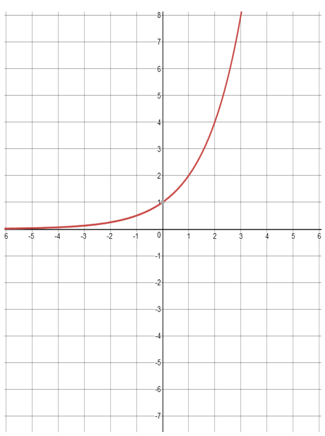
$y=-3^{x-2}$



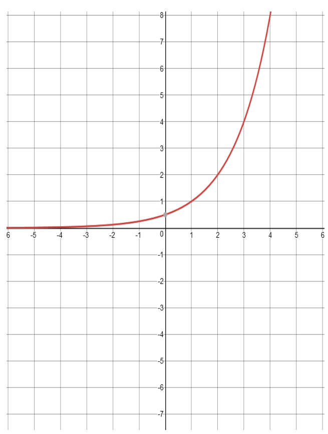
$y=-3^{x-2}+3$

3B.) (5 pts) Graph using transformations: $y=2^{1-x}-4$. Start with the base graph $y=2^x$ and then graph each successive transformation. The final graph will be your graph of $y=2^{1-x}-4$.

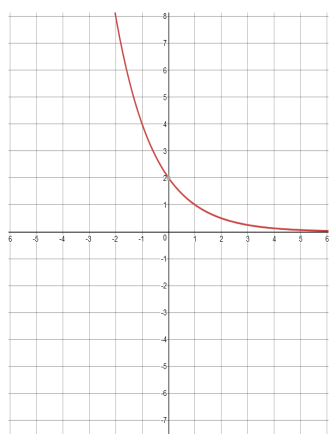
$$y=2^{1-x}-4 \quad \Rightarrow \quad y=2^{-x+1}-4 \quad \Rightarrow \quad y=2^{-(x-1)}-4$$



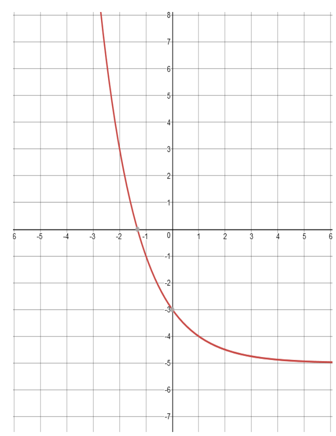
$y=2^x$



$y=2^{x-1}$



$y=2^{-(x-1)}$



$y=2^{-(x-1)}-4$

4A.) (4 pts) Solve for x: $\left(2^{2x}\cdot 2^{2x}\right)^{x-1}=8$

4A. $x=-\frac{1}{2},\ x=\frac{3}{2}$

$$\left(2^{2x+2x}\right)^{x-1}=8$$

$$\left(2^{4x}\right)^{x-1}=8$$

$$2^{4x^2-4x}=2^3$$

$$4x^2-4x=3$$

$$4x^2-4x-3=0$$

$$(2x+1)(2x-3)=0$$

$$x=-\frac{1}{2},\ x=\frac{3}{2}$$

4B.) (4 pts) Solve for x: $\left(\frac{3^{7x}}{3^{4x}}\right)^{x-1} = 729$

4B. $x = 2,\ x = -1$

$$\left(3^{7x-4x}\right)^{x-1} = 729$$

$$\left(3^{3x}\right)^{x-1} = 729$$

$$3^{3x^2-3x} = 3^6$$

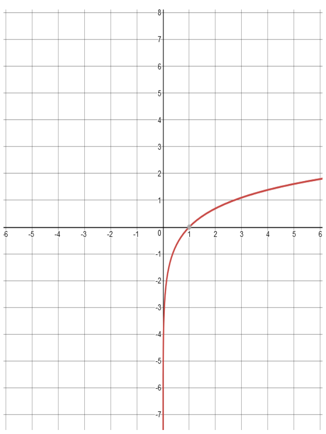
$$3x^2 - 3x = 6$$

$$3x^2 - 3x - 6 = 0$$

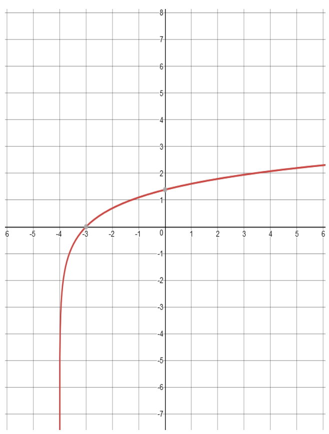
$$(3x-6)(x+1) = 0$$

$$x = 2,\ x = -1$$

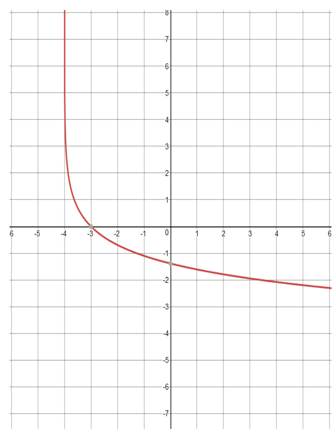
5A.) (6 pts) Graph using transformations: $y = -\ln(x+4)$. Start with the base graph $y = \ln x$ and then graph each successive transformation. The final graph will be your graph of $y = -\ln(x+4)$. Indicate the domain and vertical asymptote of your final graph.



$y = \ln x$



$y = \ln(x+4)$

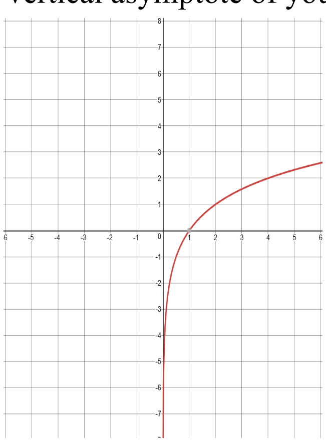


$y = -\ln(x+4)$

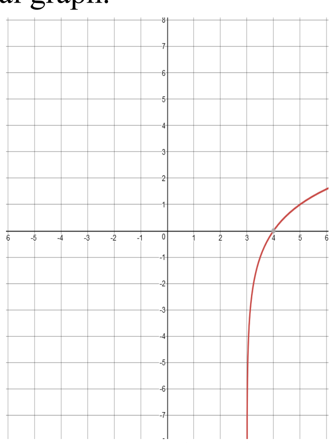
x-intercept: $x = -3$ Domain: $(-4, \infty)$ Vertical Asymptote: $x = -4$

5B.) (6 pts) Graph using transformations: $y = \log_2(3-x)$. Start with the base graph $y = \log_2 x$ and then graph each successive transformation. The final graph will be your graph of $y = \log_2(3-x)$. Indicate the domain and vertical asymptote of your final graph.

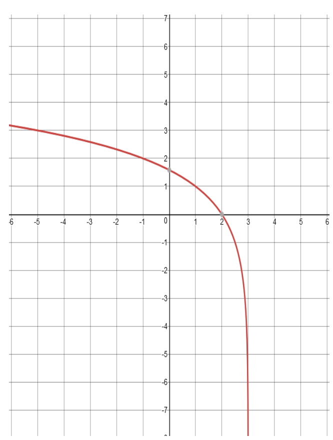
$$y = \log_2(3-x)$$
$$y = \log_2(-x+3)$$
$$y = \log_2(-(x-3))$$



$y = \log_2 x$



$y = \log_2(x-3)$



$y = \log_2(-(x-3))$

x-intercept: $x = 2$ Domain: $(-\infty, 3)$ Vertical Asymptote: $x = 3$

6A.) (5 pts) Use properties of logarithms to expand

6A. $\log_4(x-4)+\frac{1}{5}\log_4(2x-3)-1-\frac{1}{2}\log_4(4x-7)$

The logarithmic expression as much as possible.
Where possible, evaluate logarithmic expressions.

$\log_4\left[\frac{(x-4)\cdot\sqrt[5]{2x-3}}{4\sqrt{4x-7}}\right]$

$\log_4(x-4)(2x-3)^{\frac{1}{5}}-\log_4 4(4x-7)^{\frac{1}{2}}$

$\log_4(x-4)+\log_4(2x-3)^{\frac{1}{5}}-\left(\log_4 4+\log_4(4x-7)^{\frac{1}{2}}\right)$

$\log_4(x-4)+\frac{1}{5}\log_4(2x-3)-1-\frac{1}{2}\log_4(4x-7)$

6B.) (5 pts) Use properties of logarithms to expand

6B. $1+3\ln(x-2)-\ln x-\frac{1}{4}\ln(4-x)$

The logarithmic expression as much as possible.
Where possible, evaluate logarithmic expressions.

$\ln\left[\frac{(e)\cdot(x-2)^3}{x\cdot\sqrt[4]{4-x}}\right]$

$\ln(e)(x-2)^3-\ln x\cdot(4-x)^{\frac{1}{4}}$

$\ln(e)+\ln(x-2)^3-\left(\ln x+\ln(4-x)^{\frac{1}{4}}\right)$

$1+3\ln(x-2)-\ln x-\frac{1}{4}\ln(4-x)$

7A.) (4 pts) Use properties of logarithms to condense the logarithmic

7A. $\log_2\frac{5}{2x^2}$

expression. Write the expression as a single logarithm whose coefficient is 1. Fully simplify your answer:

$\log_2(x-6)+\log_2 5-\log_2(2x^3-12x^2)$

$\log_2 5(x-6)-\log_2 2x^2(x-6)$

$\log_2\frac{5(x-6)}{2x^2(x-6)}$

$\log_2\frac{5}{2x^2}$

7B.) (4 pts) Use properties of logarithms to condense the logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Fully simplify your answer:

7B. $\log\frac{x+2}{x+1}$

$\log\left(x^2+3x+2\right)-2\log(x+1)$

$$\log(x+1)(x+2)-\log(x+1)^2$$

$$\log\frac{(x+1)(x+2)}{(x+1)^2}$$

$$\log\frac{x+2}{x+1}$$

8A.) (5 pts) Solve: $\log_2(x-6)+\log_2(x-4)-\log_2 x=2$

8A. $x=12$

$$\log_2\frac{(x-6)(x-4)}{x}=2$$

$$2^2=\frac{(x-6)(x-4)}{x}$$

$$\frac{4}{1}=\frac{(x-6)(x-4)}{x}$$

$$4x=x^2-10x+24$$

$$0=x^2-14x+24$$

$$0=(x-2)(x-12)$$

~~$x=2$~~, $x=12$

$x=2$ does not work because it makes the number inside the log negative. This does not fit the domain for logs. Number inside must be great than 0.

8B.) (5 pts) Solve: $\ln x+\ln(x-2)-\ln(x+4)=0$

8B. $x=4$

$$\ln\frac{x(x-2)}{x+4}=0$$

$$e^0=\frac{x(x-2)}{x+4}$$

$$1=\frac{x(x-2)}{x+4}$$

$$x+4=x^2-2x$$

$$0=x^2-3x-4$$

$$0=(x+1)(x-4)$$

~~$x=-1$~~, $x=4$

$x=-1$ does not work because it makes the number inside the log negative. This does not fit the domain for logs. Number inside must be great than 0.

9A.) (4 pts) Solve for x: $3^{x-5} = 2^{4-x}$. You may write your answer in terms of logarithms.

9A. $x = \dfrac{4\ln 2 + 5\ln 3}{\ln 3 + \ln 2}$

$$\ln\left(3^{x-5}\right) = \ln\left(2^{4-x}\right)$$
$$(x-5)\ln 3 = (4-x)\ln 2$$
$$x\ln 3 - 5\ln 3 = 4\ln 2 - x\ln 2$$
$$x\ln 3 + x\ln 2 = 4\ln 2 + 5\ln 3$$
$$x(\ln 3 + \ln 2) = 4\ln 2 + 5\ln 3$$
$$x = \frac{4\ln 2 + 5\ln 3}{\ln 3 + \ln 2}$$

9B.) (4 pts) Solve for x: $5^{-x} = 4^{x+3}$. You may write your answer in terms of logarithms.

9B. $x = \dfrac{3\ln 4}{-\ln 4 - \ln 5}$ *or* $\dfrac{-3\ln 4}{\ln 4 + \ln 5}$

$$\ln\left(5^{-x}\right) = \ln\left(4^{x+3}\right)$$
$$-x\ln 5 = (x+3)\ln 4$$
$$-x\ln 5 = x\ln 4 + 3\ln 4$$
$$-x\ln 5 - x\ln 4 = 3\ln 4$$
$$x(-\ln 4 - \ln 5) = 3\ln 4$$
$$x = \frac{3\ln 4}{-\ln 4 - \ln 5}$$

10A.) (4 pts) Sodium-24, an isotope, has a half-life of 15 hours. How much of a 7 gram sample remains after 11 hours?

10A. 4.21 grams

$$k = \frac{-\ln 2}{15} = -0.0462$$
$$A(t) = 7e^{-0.0462t}$$
$$A(11) = 7e^{-0.0462(11)}$$
$$A(11) = 7e^{-0.5082}$$
$$A(11) \approx 4.21$$

10B.) (4 pts) An alien radioactive isotope has a half-life of 238 years. How much of a 8 kilogram sample remains after 100 years?

10B. 5.99 kg

$$k = \frac{-\ln 2}{238} = -0.0029$$
$$A(t) = 8e^{-0.0029t}$$
$$A(100) = 8e^{-0.0029(100)}$$
$$A(100) = 8e^{-0.29}$$
$$A(100) \approx 5.99$$

11A.) (4 pts) Which of the following options results in a higher amount? 11A. B

(Answer A or B)

A: \$1000 invested at 8% compounded seminannually for 3 years

B: \$1000 invested at 7.9% compounded continuously for 3 years

Option A: $A = 1000\left(1+\frac{0.08}{2}\right)^{2(3)} = 1000(1.04)^6 = \$1265.32$

Option B: $A = 1000e^{0.079(3)} = 1000e^{0.237} = \$1267.44$

11B.) (4 pts) Which of the following options results in a higher amount? 11B. A

(Answer A or B)

A: \$50 invested at 6% compounded monthly for 3 years

B: \$50 invested at 5.9% compounded continuously for 3 years

Option A: $A = 50\left(1+\frac{0.06}{12}\right)^{12(3)} = 50(1.005)^{36} = \$59.83$

Option B: $A = 50e^{0.059(3)} = 50e^{0.177} = \$59.68$

12A.) (4 pts) Las Vegas began with 2 Dunkin Donuts stores. It is estimated that 2.25 years later there will be 14 stores.

i.) Find the exponential growth function that describes the given information. i. $A(t) = 2e^{0.8648t}$

$$
\begin{aligned}
&A = A_0 e^{kt} \\
&14 = 2e^{k(2.25)} \\
&7 = e^{2.25k} \\
&\ln 7 = \ln e^{2.25k} \\
&\ln 7 = 2.25k \\
&k = \frac{\ln 7}{2.25} = 0.8648 \\
&A(t) = 2e^{0.8648t}
\end{aligned}
$$

ii.) How many stores are estimated to be in Las Vegas after 3.5 years? (Round to the nearest whole number)

ii. 41 stores

$$A(t) = 2e^{0.8648t}$$
$$A(3.5) = 2e^{0.8648(3.5)}$$
$$A(3.5) = 2e^{3.0268}$$
$$A(3.5) = 41.26$$

12B.) (4 pts) An insect population began with 500 insects. After 23.5 days the population reached 800 insects.

i.) Find the exponential growth function that describes the given information.

i. $A(t) = 500e^{0.02t}$

$$A = A_0 e^{kt}$$
$$800 = 500e^{k(23.5)}$$
$$1.6 = e^{23.5k}$$
$$\ln 1.6 = \ln e^{23.5k}$$
$$\ln 1.6 = 23.5k$$
$$k = \frac{\ln 1.6}{23.5} = 0.0200$$
$$A(t) = 500e^{0.02t}$$

ii.) What is the insect population after 10 days? (Round to the nearest whole number)

ii. 611 insects

$$A(t) = 500e^{0.02t}$$
$$A(10) = 500e^{0.02(10)}$$
$$A(10) = 500e^{0.2}$$
$$A(10) = 610.7$$

13A.) (4 pts) Solve the system: $\begin{array}{l} x^2 + y^2 = 1 \\ y = x^2 - 1 \end{array}$. Write your answer as coordinates.

(0, –1)

(1, 0)

(–1, 0)

This can be done by either substitution or elimination:

Substitution:

$$x^2 + (x^2 - 1)^2 = 1$$
$$x^2 + x^4 - 2x^2 + 1 = 1$$
$$x^4 - x^2 = 0$$
$$x^2(x^2 - 1) = 0$$
$$x^2 = 0 \quad or \quad x^2 - 1 = 0$$
$$x = 0 \quad or \quad x = \pm 1$$

Now plug x values into $y = x^2 - 1$:

$x = 0$	$x = 1$	$x = -1$
$y = 0^2 - 1 = -1$	$y = 1^2 - 1 = 0$	$y = (-1)^2 - 1 = 0$

Elimination:

$$x^2 + y^2 = 1 \qquad \text{(Subtract Equations)}$$
$$x^2 - y = 1$$

$$x^2 + y^2 - (x^2 - y) = 1 - 1$$
$$y^2 + y = 0$$
$$y(y + 1) = 0$$
$$y = 0 \quad or \quad y = -1$$

Now plug y values into $x^2 - y = 1$:

$y = 0$	$y = -1$
$x^2 - 0 = 1$	$x^2 - (-1) = 1$
$x = \pm 1$	$x = 0$

13B.) (4 pts) Solve the system: $\begin{array}{l} x^2 + 2y^2 = 9 \\ 4x^2 - y^2 = 0 \end{array}$. Write your answer as coordinates.

(1, 2)

(1, –2)

(–1, 2)

(–1, –2)

Elimination is the best option for this problem. Multiply the bottom equation by 2 and add the equations together.

$$x^2 + 2y^2 = 9$$
$$8x^2 - 2y^2 = 0$$
$$9x^2 + 0 = 9$$

$$x^2 = 1$$
$$x = \pm 1$$

Now plug x values into either equation:

$$4x^2 - y^2 = 0$$

$x = 1$	$x = -1$
$4(1)^2 - y^2 = 0$	$4(-1)^2 - y^2 = 0$
$4 = y^2$	$4 = y^2$
$y = \pm 2$	$y = \pm 2$