2.1 Functions

Domain: (input) all the x-values that make the equation defined
Defined: There is no division by zero or square roots of negative numbers
Range: (output) all y-values that a graph uses.

Function Definition: For each input (x) there can only be one output (y).

EXAMPLE: For each relation below, determine whether it is a function. Then give the domain and range for each relation.

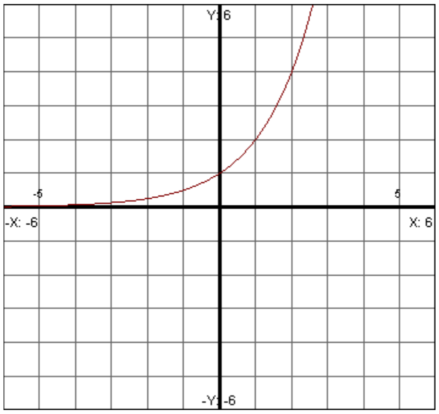
{(1, 2), (3, 7), (2, 9), (8, 11)}

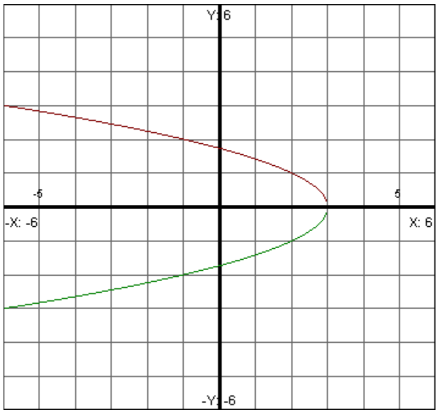
{(-3, 4), (5, 6), (7, 4), (-2, 3)}

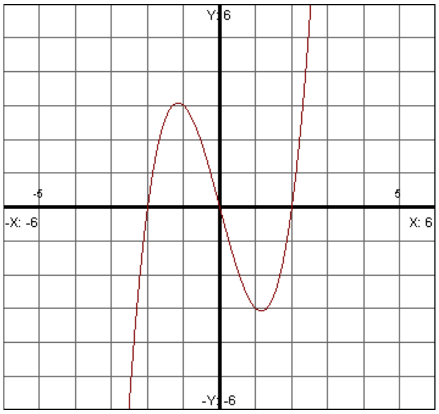
{(-2, 4), (-1, 6), (0, 3), (-2, 8)}

{(5, 3), (-2, 1), (5, 3), (9, 10)}

<u>Vertical line test</u>. If you pass an imaginary vertical line through the graph and it only intersects the graph once then it is a function. Which graphs below are functions?







EXAMPLE: Is $x^2 + 3y^2 = 7$ a function?

EXAMPLE: Is $x^2 + y^5 = 1$ a function?

Any equation that has a y raised to an even power is NOT a function.
Any equation that has a y raised to an odd power IS a function.

Function notation: $f(x)$ which means "f of x". This does not mean f times x. It means that we have a function called f which contains the variable x.

EXAMPLE: Given the function $f(x) = 2x - 5$, find the following:

a.) Find $f(3)$. Solve $f(x) = 7$.

b.) Given the function $f(x) = 2x - 5$, find $f(x+3)$

c.) Given the function $f(x) = 2x - 5$, find $f(x) + f(3)$.

d.) Given the function $f(x) = 2x - 5$, find $f(x+h)$.

EXAMPLE: Let $f(x) = \dfrac{x-4}{2x-3}$. Find the following:

a.) $f(5)$.

b.) $f(x+h)$

c.) Given $f(x) = \frac{x-4}{2x-3}$, find $f(x) + f(5)$.

Finding Domain Algebraically

EXAMPLE: Find the domain: $y = 2x - 5$

EXAMPLE: Find the domain: $y = 5x^2 - 3$

EXAMPLE: Find the domain: $y = \frac{x-3}{2x-5}$

EXAMPLE: Find the domain: $y = \sqrt{2x-5}$

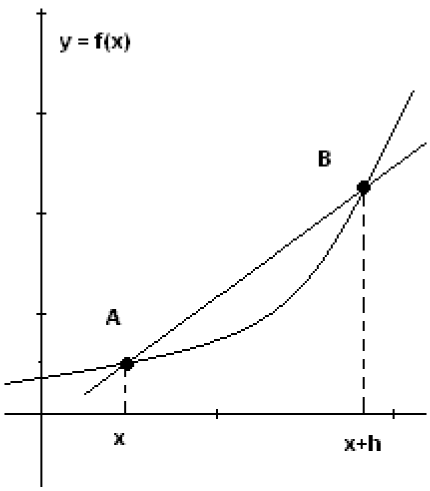
EXAMPLE: Find the domain: $y = \dfrac{7x}{\sqrt{2x-5}}$

EXAMPLE: Find the domain: $y = \dfrac{x}{\sqrt[3]{x-3}}$

EXAMPLE: Find the domain: $y = \dfrac{1}{x^2+9}$

EXAMPLE: Find the domain: $y = \dfrac{\sqrt{x-3}}{x-6}$

Difference Quotient



If we wanted to find the slope of a curved line, the only way we can do this is by estimating it with a straight line. We will start with one point and then move over by a small amount h. Now we will use the slope formula. In the picture we have two points, A and B. The coordinates for these are: $(x, f(x))$ and $(x+h, f(x+h))$.

The slope, also called the **difference quotient** is: $\dfrac{f(x+h)-f(x)}{h}$

In calculus we will try to minimize h so that it is so small that we end up at a point, which will be the exact slope of the curved line at x.

Now let's look at some examples finding the difference quotient.

EXAMPLE: Let $f(x) = 2x - 3$. Find the difference quotient.

EXAMPLE: Let $f(x) = 3x^2 - x + 1$. Find the difference quotient.

EXAMPLE: $f(x) = \dfrac{3}{x-1}$. Find the difference quotient.