4.3 Properties of Rational Functions

Rational Function: a function whose numerator and denominator are polynomials

To find the y-intercept for a rational function, put in a zero for x.

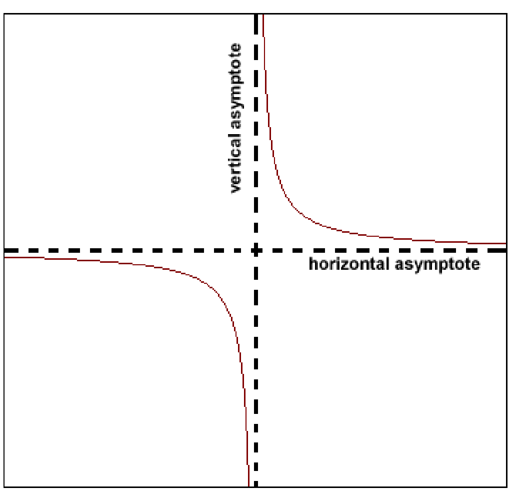
To find the x-intercept for a rational function, set the numerator equal to zero

EXAMPLE: Find the x and y intercepts for $y = \dfrac{2x-12}{x+2}$

x-int: set top equal to zero

y-int: 0 for x.

Asymptote: describes the behavior of a graph as x or y approaches infinity. There are two types of asymptotes. There is a vertical and horizontal asymptote as show in the picture below. The vertical asymptote has an equation that starts with x = since this is a vertical line. The horizontal asymptote has an equation y = since this is a horizontal line. Now we will show how you find these algebraically.



To find the vertical asymptote:
set the denominator equal to zero and solve for x.

To find the horizontal asymptote:

First we need to define some variables. Let's look at the general form of rational expression:

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + ...}{b_m x^m + b_{m-1} x^{m-1} + ...}$$

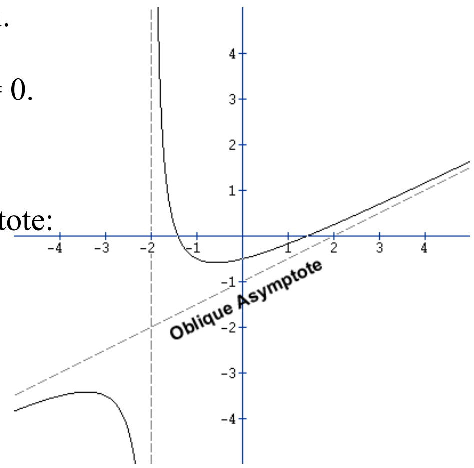
Let n be the highest power (degree) of the numerator.
Let m be the highest power (degree) of the denominator.

Let a_n be the number that comes in front of the x with the highest power in the numerator.
Let b_m be the number that comes in front of the s with the highest power in the denominator.
In order to determine the horizontal asymptote we need to look at the n and m.

1.) If n < m then the equation of the horizontal asymptote is automatically y = 0.

2.) If n = m then the equation of the horizontal asymptote is $y = \frac{a_n}{b_m}$.

3.) If n > m then there is no horizontal asymptote. There is an oblique asymptote:
You find the oblique asymptotes by using long division. More on this later.

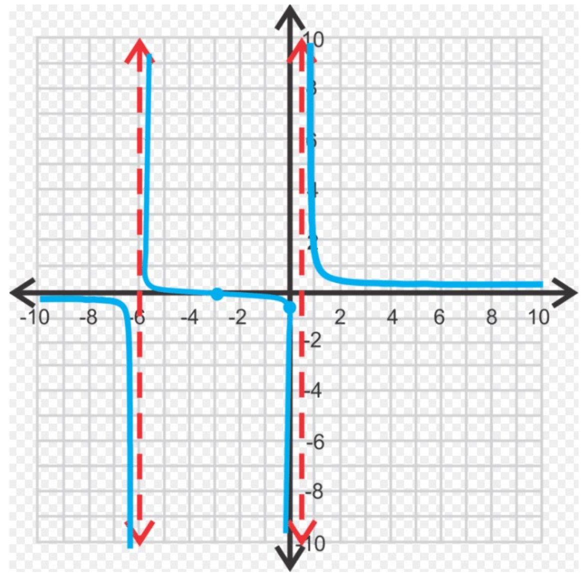


EXAMPLE: Find the asymptotes for $y = \frac{2x-12}{x+2}$

Vertical asymptote:

Horizontal asymptote:

EXAMPLE: Use the graph shown to find the following: (a) The domain and range of the function; (b) The intercepts, if any; (c) horizontal asymptotes, if any; (d) vertical asymptotes, if any; (e) oblique asymptotes, if any.



EXAMPLE: Find the intercepts and asymptotes but DO NOT GRAPH: $y = \frac{1-x^2}{x^2-5x+6}$. State the domain.

x-int: Top = 0

y-int: 0 for x

V.A. Bottom = 0

H.A. (Rules)

Domain:

EXAMPLE: Find the intercepts and asymptotes but DO NOT GRAPH: $y = \dfrac{x^2 - 3x}{2x^2 + 4x^3}$.

x-int: Top = 0

y-int: 0 for x

V.A. Bottom = 0

H.A. (Rules)

EXAMPLE: Find the asymptotes but DO NOT GRAPH: $y = \dfrac{3x^2 + 10x + 6}{x + 2}$

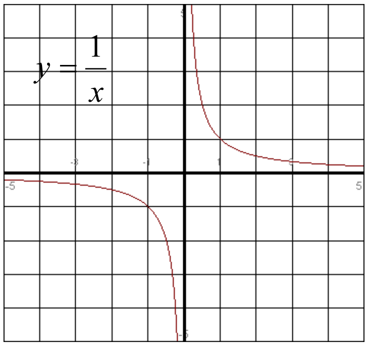
x-int: Top = 0

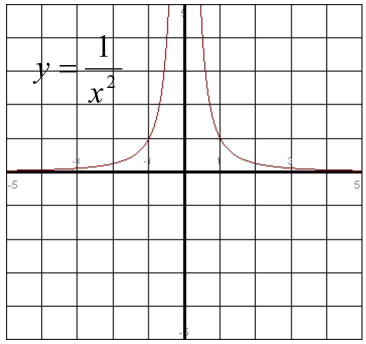
y-int: 0 for x

V.A. Bottom = 0

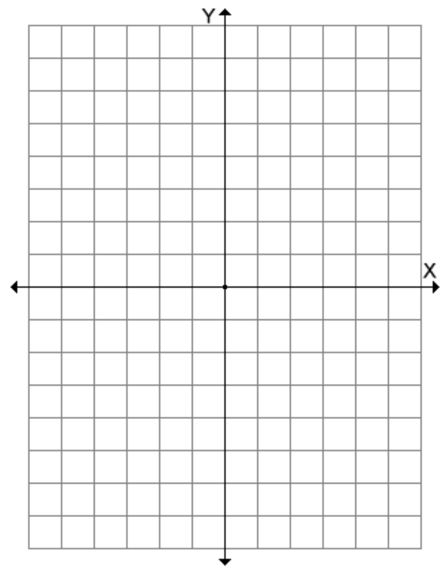
H.A. (Rules)

Next we will look at some special graphs. These are: $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$. The normal graphs look like:

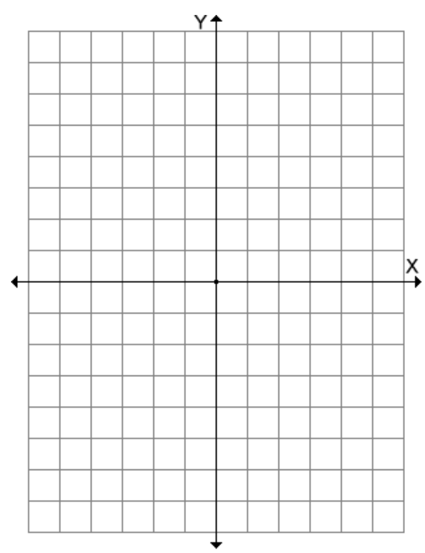




EXAMPLE: Graph $y = \frac{1}{x-3}$ using transformations.



EXAMPLE: Graph $y = \frac{-1}{x^2+6x+9}$ using transformations.



EXAMPLE: Graph $y=\dfrac{x-1}{x}$ using transformations.