

4.6 The Real Zeros of Polynomial Functions

In the previous section, we learned how to divide polynomials. Now we will expand on that and learn some new theorems.

Remainder Theorem: If $f(x)$ is divided by $x - c$, then the remainder is $f(c)$

Factor Theorem: If $f(c) = 0$ then $x - c$ is a factor of $f(x)$. Also, if $x - c$ is a factor of $f(x)$ then $f(c) = 0$.

EXAMPLE: Given $f(x) = 2x^3 - 4x^2 + 5x - 3$, find $f(2)$. Then use the Remainder Theorem to find the remainder when $f(x)$ is divided by $x - 2$. Finally use the Factor Theorem to determine whether $x - 2$ is a factor of $f(x)$. Is $x = 2$ a zero of $f(x)$?

EXAMPLE: Given $f(x) = 2x^4 + 12x^3 + 6x^2 - 5x + 75$, find $f(-5)$. Then use the Remainder Theorem to find the remainder when $f(x)$ is divided by $x + 5$. Finally, use the Factor Theorem to determine whether $x + 5$ is a factor of $f(x)$. Is $x = -5$ a zero of $f(x)$?

EXAMPLE: Solve the equation $2x^3 - 3x^2 - 11x + 6 = 0$ given that -2 is a zero of $f(x) = 2x^3 - 3x^2 - 11x + 6$.

Rational Zeros Theorem

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ be a polynomial. Then the number of possible real zeros of f is:

$$\frac{\text{factors of } a_0}{\text{factors of } a_n}$$

To review a factor is a number that evenly divides into something. For example, the factors of 6 are 1, 2 and 3.

EXAMPLE: List the possible real zeros of: $f(x) = 3x^3 - 7x^2 - 22x + 8$

EXAMPLE: Given $f(x) = x^3 + 8x^2 + 11x - 20$, a.) Use the Rational Zero Theorem to find the list of possible zeros. b.) Find the zeros using synthetic division. c.) Use the zeros to factor the polynomial.

a.) Use the Rational Zero Theorem to find the list of possible zeros.

b.) Find the zeros using synthetic division.

c.) Use the zeros to factor the polynomial.

EXAMPLE: Given $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$, a.) Use the Rational Zero Theorem to find the list of possible zeros. b.) Find the zeros using synthetic division. c.) Use the zeros to factor the polynomial.

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b.) Find the zeros using synthetic division.

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EXAMPLE: Given $f(x) = x^4 + 4x^3 + 3x^2 - 4x - 4$, a.) Use the Rational Zero Theorem to find the list of possible zeros. b.) Find the zeros using synthetic division. c.) Use the zeros to factor the polynomial.

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