4.7 The Complex Zeros of Polynomial Functions

This section is similar to the last one except we will be finding complex, or imaginary zeros. Complex, or imaginary zeros always come in conjugate pairs. For example, if it is known that one zero is 7 + 3i, the other one must be 7 – 3i.

EXAMPLE: If a degree 6 polynomial has complex zeros i, 3 – 2i, and -2 + i, what must the other zeros be?

EXAMPLE: Form a degree 4 polynomial with real number coefficients if it has complex zeros of i and 1 + 2i.

EXAMPLE: The polynomial $g(x) = x^3 + 3x^2 + 25x + 75$ has one zero of -5i. Use this to find the other zeros.

EXAMPLE: The polynomial $g(x) = x^4 - 7x^3 + 14x^2 - 38x - 60$ has one zero of 1 + 3i. Use this to find the other zeros.

EXAMPLE: The polynomial $g(x) = 6x^4 - 7x^3 + 93x^2 - 112x - 48$ has one zero of 4i. Use this to find the other zeros.

EXAMPLE: Given $f(-5) = 0$, find the complex zeros of $f(x) = x^3 + 13x^2 + 57x + 85$.

EXAMPLE: Given $f(x) = 2x^4 + x^3 - 35x^2 - 113x + 65$, a.) Use the Rational Zero Theorem to find the list of possible zeros. b.) Find the zeros using synthetic division given $f(5) = 0$ and $f\left(\frac{1}{2}\right) = 0$. c.) Use all the zeros to factor the polynomial.