5.2 One to One Functions; Inverse Functions

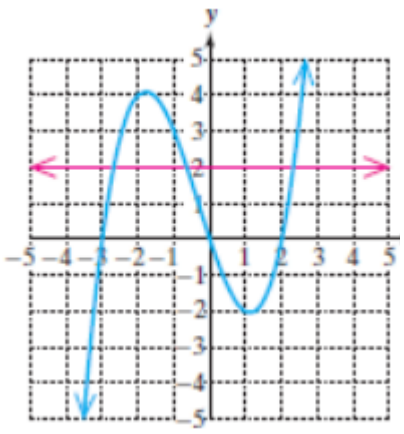
One-To-One Function

A function f is a one-to-one function if for a and b in the domain of f, if $a \neq b$, then $f(a) \neq f(b)$, or equivalently, if $f(a) = f(b)$ then $a = b$. In other words, for each y value there can only be one x value.

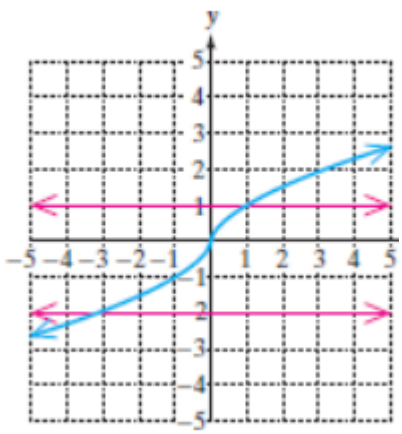
Horizontal Line Test

If you pass a horizontal line and it hits the graph at only one place, then it is one-to-one.
EXAMPLE: Use the horizontal line test to determine if the graph below defines y as a one-to-one function.

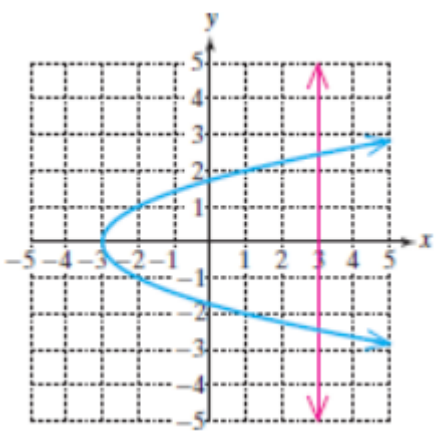
a.



b.



c.

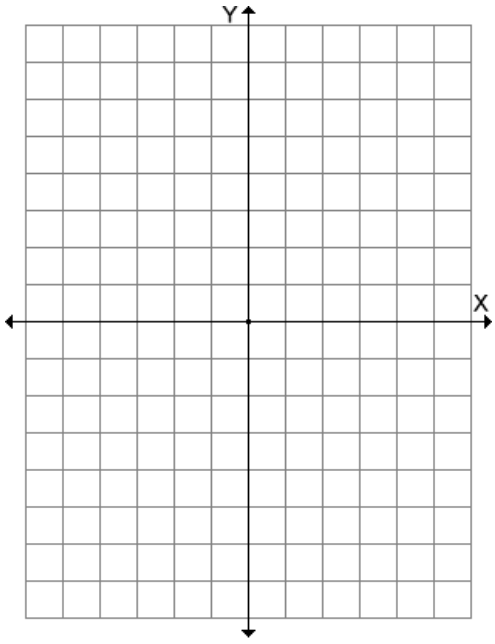


Inverse Functions
Let f be a one-to-one function. Then g is the inverse of f if the following conditions are both true:
$(f \circ g)(x) = x$ and $(g \circ f)(x) = x$

We just talked about the fact that the x and y coordinates are reversed on the graph. This means the following:
If $f(x) = y$, then $f^{-1}(y) = x$.

EXAMPLE: If you are given $f(10) = 14$, find $f^{-1}(14)$.

EXAMPLE: Given $f(x) = 2x - 1$ and $g(x) = \frac{1}{2}x + \frac{1}{2}$ verify that they are inverses. Then graph each line.



Notation to write "the inverse of $f(x)$" is $f^{-1}(x)$. This does not mean f raised to the negative one power. It just means we have the inverse of $f(x)$.

EXAMPLE: Verify the following are inverses: $f(x) = \sqrt{x-3}$ and $f^{-1}(x) = x^2 + 3$

How to find an inverse algebraically:

Step 1: Replace $f(x)$ with y.
Step 2: Switch x and y.
Step 3: Solve for y.
Step 4: Replace y with $f^{-1}(x)$.

EXAMPLE: Given $f(x) = 2x - 5$ find $f^{-1}(x)$. Then verify your answer is correct.

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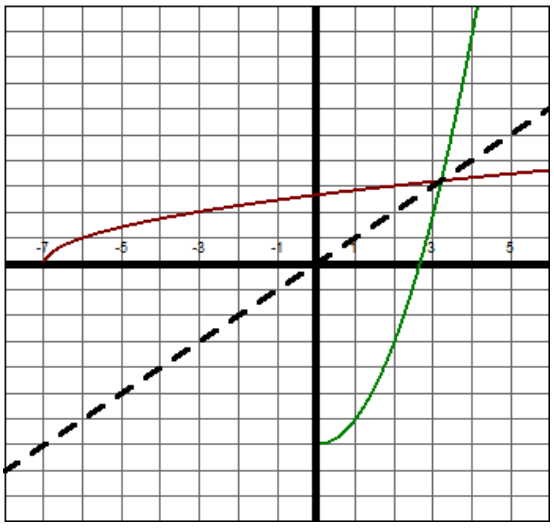
EXAMPLE: Given $f(x)=\sqrt{x+7}$ find $f^{-1}(x)$. Then verify your answer is correct.



EXAMPLE: Given $f(x)=\dfrac{2x-3}{x+4}$ find $f^{-1}(x)$. You do not need to verify this one.

EXAMPLE: Given $f(x)=\dfrac{3x-5}{2x-3}$ find $f^{-1}(x)$. You do not need to verify this one.