5.4 Logarithmic Functions

In a previous section we looked at inverses. In order to find a inverse we need to switch *x* and *y*.

Suppose we wanted to find the inverse of our exponent function, $y = b^x$. First we need to switch *x* and *y*. We will get $x = b^y$. How do we solve for *y*? This is where we need logarithms, which are a way to solve for an exponent.

With logarithms there are two forms: exponential and logarithmic.

Exponential form: $x = b^y$ **Logarithmic form**: $y = \log_b x$

EXAMPLE: Change $7 = \log_m 5$ into exponential form.

EXAMPLE: Change $\log_c 6 = 8$ into exponential form.

EXAMPLE: Change $q = 1.4^5$ into logarithmic form.

EXAMPLE: Change $2^d = 8$ into logarithmic form.

EXAMPLE: Change $x = e^y$ into logarithmic form.

EXAMPLE: Find the value of $\log 5$ on a calculator and round to the nearest thousandth.

EXAMPLE: Find the value of $\ln 7$ on a calculator and round to the nearest hundredth.

Equal Bases Property

If $a^u = a^v$ then u = v.

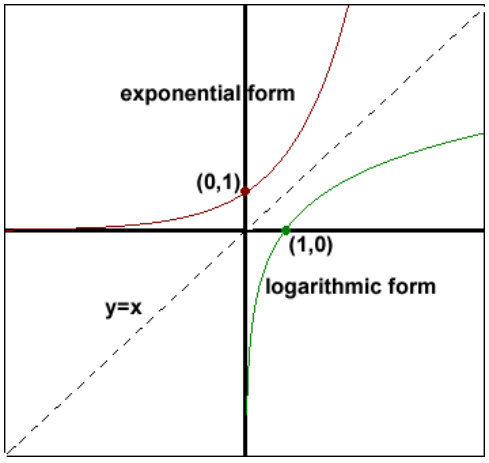
EXAMPLE: Find the exact value of $\log_4 64$.

EXAMPLE: Find the exact value of $\log\frac{1}{10000}$.

EXAMPLE: Find the value of $\log 0$ on a calculator and round to the nearest hundredth.

Let's try and draw a graph of $y = \log_b x$. In order to do this, we need to first draw the graph of $x = b^y$.

Key Points: (1, 0) and (*b*, 1)



Domain of $y = \log_b x$ **is x > 0**. Notice that zero is not included since this is a vertical asymptote. Also notice that it doesn't matter what *b* is.

EXAMPLE: Find the domain of $y = \ln(2x - 7)$ and write your answer in interval notation.

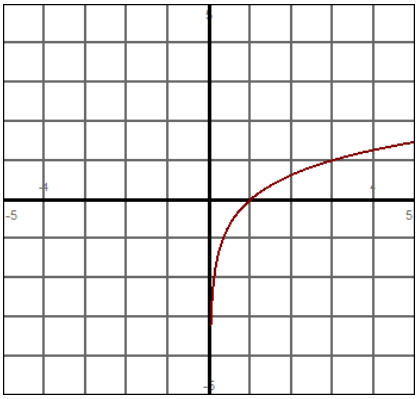
EXAMPLE: Find the domain of $y = \log_3(6 - x)$ and write your answer in interval notation.

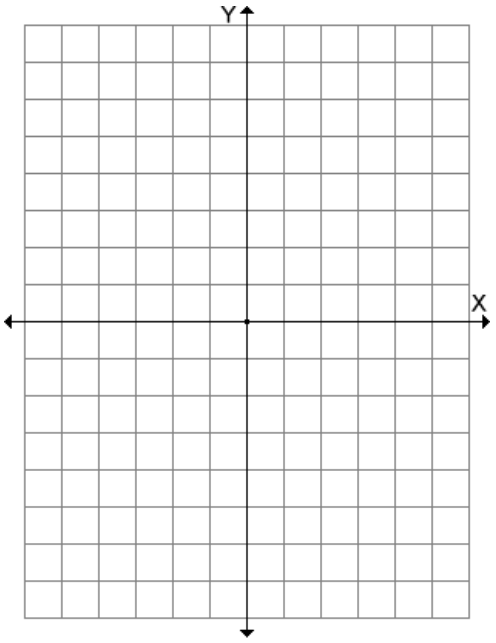
EXAMPLE: Find the domain of $y = \log_2\left(\frac{x+2}{2-x}\right)$ and write your answer in interval notation.

$x + 2$			
$2 - x$			

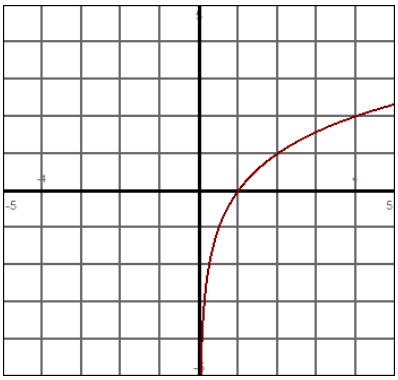
-2 2

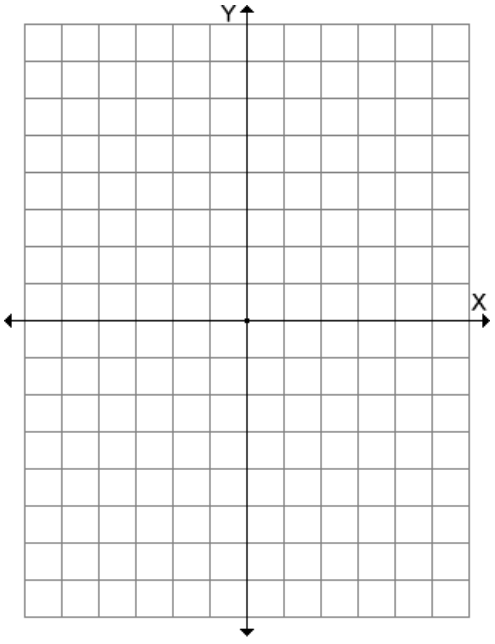
EXAMPLE: Graph using transformations: $y = \log_3(x-4)$ and identify the x-intercept.



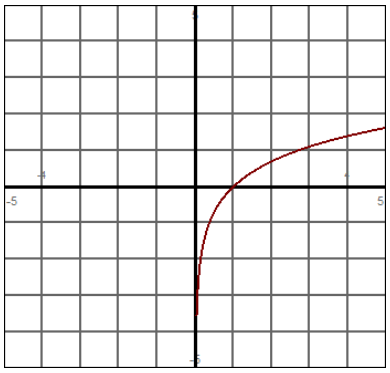


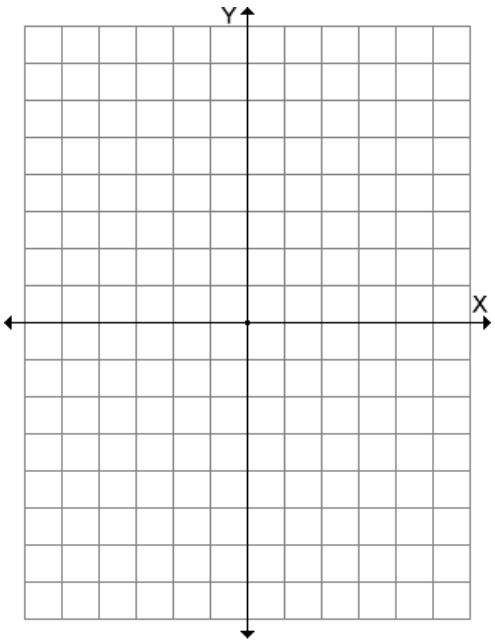
EXAMPLE: Graph using transformations: $y = -\log_2(x+3)$ and identify the x-intercept.



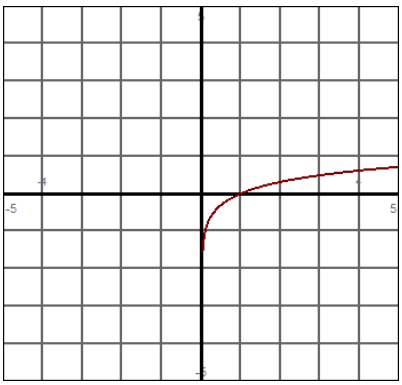


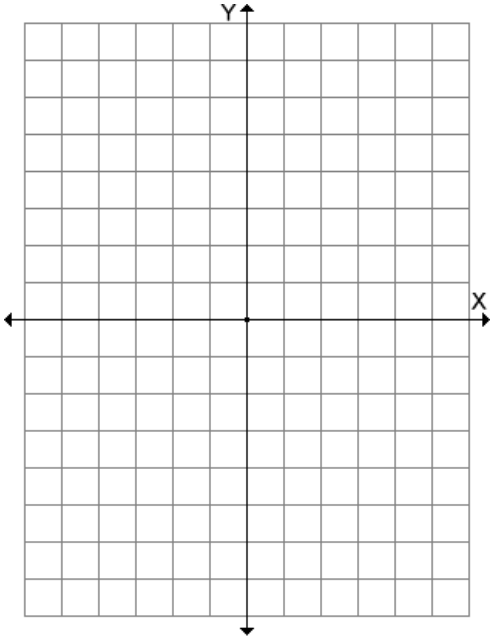
EXAMPLE: Graph using transformations: $y = \ln(1 - x)$ and identify the x-intercept.





EXAMPLE: Graph using transformations: $y = -\log\left(-\left(x + 2\right)\right)$ and identify the x-intercept.





Solving Logarithmic Equations by Changing into Exponential Form

EXAMPLE: Solve: $\log_5 x = 3$.

EXAMPLE: Solve: $\log_x\left(\frac{1}{8}\right) = 3$.

EXAMPLE: Solve: $\log_4\left(x^2 + 9\right) = 2$.

EXAMPLE: Solve: $\log_5(4x+5)=2$.

EXAMPLE: Solve: $\log_{1/2}(3x-7)=3$.

EXAMPLE: Solve: $e^{x+5}=4$.