5.5 Logarithmic Properties

Properties of Logarithms

1.) $\log_b 1 = 0$ Example: $\log_3 1 = 0$, $\ln 1 = 0$

2.) $\log_b b = 1$ Example: $\log_2 2 = 1$, $\ln e = 1$, $\log 10 = 1$

3.) $b^{\log_b M} = M$ Example; $2^{\log_2 5} = 5$, $5^{\log_5 \pi} = \pi$

4.) $\log_b b^r = r$ Example: $\log_3 3^7 = 7$, $\log_2 2^5$

5.) $\log_b M^r = r \cdot \log_b M$ Example: $\log_3 5^8 = 8 \cdot \log_3 5$

6.) $\log_b (M \cdot N) = \log_b M + \log_b N$ Example: $\log_2 (3 \cdot 5) = \log_2 3 + \log_2 5$

7.) $\log_b \left(\frac{M}{N} \right) = \log_b M - \log_b N$ Example: $\log_3 \left(\frac{24}{6} \right) = \log_3 24 - \log_3 6$

EXAMPLE: Find the exact value using logarithm properties: $\log_3 \sqrt[9]{3}$.

EXAMPLE: Find the exact value using logarithm properties: $e^{\ln 6}$.

EXAMPLE: Find the exact value using logarithm properties: $\log_2\left[\log_{1/2}\left(\frac{1}{4}\right)\right]$.

EXAMPLE: Find the exact value using logarithm properties: $5^{\log_5 6+\log_5 7}$.

EXAMPLE: Express $\log_9 x^2 \cdot \sqrt{3x-5}$ as a sum or difference of logarithms. Express powers as factors.

EXAMPLE: Express $\ln\frac{(x+5)^4}{x^3}$ as a sum or difference of logarithms. Express powers as factors.

EXAMPLE: Express $\log_4 \dfrac{(x-5)^5 \cdot \sqrt[3]{x-2}}{(x-1)^4}$ as a sum or difference of logarithms. Express powers as factors.

EXAMPLE: Express $\ln\left[\dfrac{x^2-5x+6}{(x+2)^3}\right]^{\frac{1}{4}}$ as a sum or difference of logarithms. Express powers as factors.

EXAMPLE: Express $\log_5 \dfrac{2x^4(x-4)}{5(x-3)^5}$ as a sum or difference of logarithms. Express powers as factors.

EXAMPLE: Express $\dfrac{1}{2}\cdot\log_6 x + 2\cdot\log_6(x-2)$ as a single logarithm without negative exponents.

EXAMPLE: Express $\ln(x^2 - 1) - 2 \cdot \ln(x + 1)$ as a single logarithm without negative exponents.

EXAMPLE: Express $-2 \cdot \log_3 x + \log_3(x^2 + 2) - \log_3 4 + 3 \cdot \log_3 x + \log_3 8$ as a single logarithm without negative exponents.

Change-of-Base Formula

In the last section we mentioned that the only type of logs we can do on our calculator is base 10 and base e. This formula will allow us to find the numerical value of a log with ANY base.

$\log_a M = \dfrac{\log_b M}{\log_b a}$ In this formula the b can be any base we want.

EXAMPLE: Use the Change-of-Base Formula to calculate $\log_5 8$ to two decimal places.

EXAMPLE: Use the Change-of-Base Formula to calculate $\log_3 8 \cdot \log_8 9$ to two decimal places.