5.8 Exponential Growth and Decay; Newton's Law of Cooling

There are many applications to exponential functions that we will focus on in this section. First let's look at the exponential model.

Exponential Growth / Decay Model

$$A(t) = A_o e^{k \cdot t}$$

$A(t)$ = current population, A_o = original population, k = growth or decay constant
t = time measured in any unit.

EXAMPLE: At the start of the experiment there are 125 cells. Three hours later there are 235 cells.

a.) What is the exponential growth formula?

b.) How many cells are present after 5 hours?

c.) How long (rounded to the nearest hour) will it take the population to reach 442 cells?

EXAMPLE: At the start of the experiment there are 1000 bacteria. After 4 hours the population doubled.

a.) What is the exponential growth formula?

b.) How many bacteria are present after 6 hours?

EXAMPLE: Complete the table:

2000 pop (in millions)	Projected 2011 Pop (millions)	Projected Growth Rate
53.4		0.0108

Half-life Problems

The next problems will focus on types of decay. We will first look at half-life problems. Half-life is the amount of time it takes half of a substance to decay. The problems we will do will contain this information. We will still use the same exponential model, except that half-life problems have a special formula to solve for *k*.

Decay constant for a half-life problem: $k = \dfrac{-\ln 2}{half\ \ life}$.

EXAMPLE: The half-life of strontium-90 is 28 years.

a.) What is the decay function if the initial sample was 10 grams?

b.) How much of a 10 gram sample is left after 11 years?

EXAMPLE: The half-life of radium-226 is 1620 years. How much of a 2 gram sample is left after 1000 years?

Newton's Law of Cooling:

$u(t) = T + (u_o - T)e^{kt}$, where $k < 0$.

$u(t)$ = final temperature after cooling T = temperature of the surrounding atmosphere
u_o = initial temperature before cooling k = cooling constant (must be negative) t = time

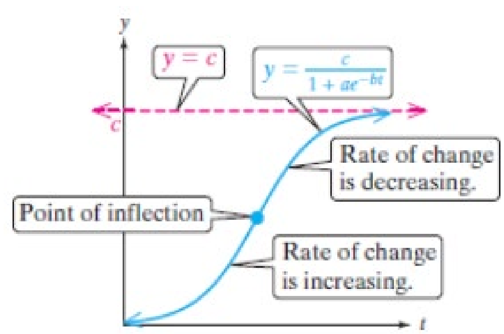
EXAMPLE: A pizza removed from the oven has a temperature of $450^\circ F$. It is left sitting in a room that has a temperature of $70^\circ F$. After 5 minutes the temperature of the pizza is $300^\circ F$. What is the temperature of the pizza after 20 minutes? When will the temperature of the pizza be $140^\circ F$?

EXAMPLE: A thermometer reading $72^\circ F$ is placed in a refrigerator where the temperature is a constant $38^\circ F$. After 2 minutes the thermometer reads $60^\circ F$. What will it read after 7 minutes? How long will it take before thermometer reads $39^\circ F$?

Logistic Growth Model

If you had a Petri dish with bacteria inside, the bacterial would grow exponentially, but not forever. Once all the food is used up the growth will level off. So a logistic growth model (listed below) best describes this situation. Use this model when there is a restriction to growth

$y = \dfrac{c}{1 + ae^{-bt}}$ where *a*, *b*, *c* are constants.



As in the figure, the c value indicates what the growth will level off at c. (horizontal asymptote). The *c* is also called the **limiting factor**.
The term containing *a* and *b* will go to zero as *t* approaches infinity.
The *b* term is the growth rate.

EXAMPLE: The population of Canada P(t) (in millions) since January 1, 1900, can be approximated by

$P(t) = \dfrac{55.1}{1+9.6e^{-0.02515t}}$ where t is the number of years since January, 1, 1900.

a.) Evaluate $P(0)$ and evaluate its meaning.

b.) What is the growth rate of the Canadian population?

c.) Use the function to approximate the Canadian population on January 1, 2015. Round to the nearest tenth of a million.

d.) From the model, during which year would the Canadian population reach 45 million?