# 11.6 Systems of Nonlinear Equations: Two Variables

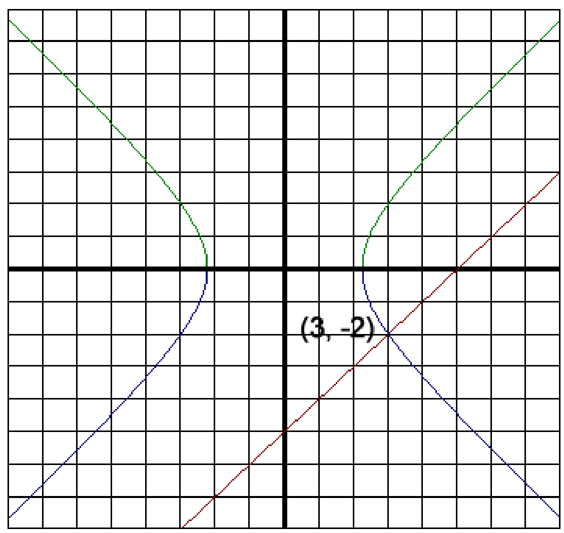
In this section we will still be solving equations using the elimination and substitution method, but the equations will not be linear in style.

**Nonlinear Systems of Equations**

EXAMPLE: Solve the system: $\begin{array}{l} y = x - 5 \\ x^2 - y^2 = 5 \end{array}$

You will need to decide in these problems which method is easier to use: elimination or substitution. In this case substitution will be the easiest since one of the variables has already been solved for. We will substitute the top equation into the bottom one. We will replace the y in the bottom equation and replace it with x – 5. You will have: $x^2 - (x-5)^2 = 5$. Now everything is in terms of a single variable. We need to expand this to: $x^2 - (x^2 - 10x + 25) = 5$. Now distribute the negative: $x^2 - x^2 + 10x - 25 = 5$. We cancel the x squared terms to get $10x - 25 = 5$. Solving this we get x = 3. We need to also find y so we can substitute a 3 for x in the first equation: y = 3 – 5, so y = -2. As a point this would be written as (3, -2).

You are not expected to know how to graph the second equation yet but I want to show it to you anyway so you can see what it is you are actually doing. You are still finding the intersection of two graphs. The first one is a line and the second graph is called a hyperbola, which we will focus on in chapter 9. Notice in this graph that these intersect at (3, -2) which we found algebraically.




EXAMPLE: Solve the system: $\begin{array}{l} xy = -2 \\ x + 2y = 0 \end{array}$

We will use substitution for this one. Let's solve the bottom equation for x. This way we won't get fractions. Solving for x in the bottom equation will give us $x = -2y$. Now we will substitute this into the top equation. Replace the x with -2y. Then you will have the equation: $(-2y)y = -2$. This will give us $-2y^2 = -2$. Divide both sides by -2 to get $y^2 = 1$. After taking the square root of both sides you will get $y = \pm 1$.

Since we get two different values for y this means we will get two different values for x. We previously solve for x and we got $x = -2y$. When y = 1 we get: x = -2(1) = -2. When y = -1 we get x = -2(-1) = 2. So we get two points as our answer: (-2, 1) and (2, -1). Make sure you put the correct y value with each x value.

EXAMPLE: Solve the system: 
$$x^2 - y^2 = 0$$
$$2x^2 + 3y^2 = 5$$

It would be difficult to solve for x or y on this one so it will be easier to solve this one using elimination. I can choose to eliminate either the x or the y. I will eliminate the y. All I need to do is multiply the first equation by 3. I will get: $\begin{array}{l} 3x^2 - 3y^2 = 0 \\ 2x^2 + 3y^2 = 5 \end{array}$. Now add these equations together to get: $5x^2 = 5$. Divide both sides by 5 to get $x^2 = 1$. We will get: $x = \pm 1$. Now substitute these values back into either the first or second equation. Let's use the first equation. Let's start with x = 1. We will put a 1 in for x in the first equation: $1^2 - y^2 = 0$. You will get $y^2 = 1$, so $y = \pm 1$. Now we will use x = -1. We will put a -1 in for x in the first equation: $(-1)^2 - y^2 = 0$. You will get $y^2 = 1$, so $y = \pm 1$.

So we get four points as our solution: (1, 1), (1, -1), (-1, 1), (-1, -1).

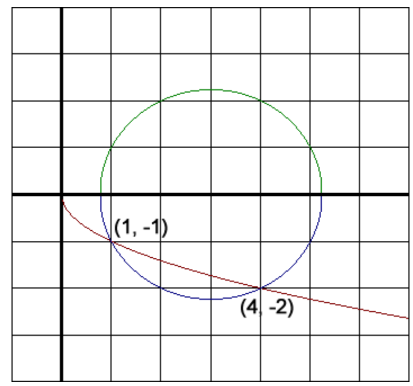
EXAMPLE: Solve the system: 
$$x^2 + y^2 = 10$$
$$2x^2 - y = -1$$

There is not a typo in this one. The second equation does not have the y as being squared. Again I think it will be easier to solve this one using elimination. I will eliminate the x. I need to do is multiply the first equation by -2. I will get: $\begin{array}{l} -2x^2 - 2y^2 = -20 \\ 2x^2 - y = -1 \end{array}$. Now add these equations together to get: $-2y^2 - y = -21$. Notice I can't combine the two terms on the left side of the equals sign since these are not like terms. I need to set this equal to zero, so I will add 21 to both sides: $-2y^2 - y + 21 = 0$. Now I will multiply both sides by -1 to make it easier to factor: $2y^2 + y - 21 = 0$. Now factor it using whichever method is easiest. You will get: $(2y + 7)(y - 3) = 0$. Solving this will give us two answers for y: $y = -\frac{7}{2}$ and $y = 3$. We can put these into the second equation to get x. Let's start with $y = -\frac{7}{2}$. We will put this into the second equation for y: $2x^2 - \left(-\frac{7}{2}\right) = -1$, or $2x^2 + \frac{7}{2} = -1$. Now solve. We need to subtract seven-halves from both sides: $2x^2 = -\frac{9}{2}$. We can multiply both sides of the equation by one-half to get: $x^2 = -\frac{9}{4}$. If you try to take the square root of both sides you will get an imaginary number. We do not include imaginary number solutions in this section. We only want to find the actual place the graphs meet. So we know that $y = -\frac{7}{2}$ is not going to give us an x. Let's now try y = 3. We will put this into the second equation for y: $2x^2 - 3 = -1$. Add 3 to both sides to get: $2x^2 = 2$. Divide both sides by 2 to get: $x^2 = 1$. Then we get $x = \pm 1$.

So we will get two places these graphs intersect: (1, 3) and (-1, 3).

EXAMPLE: Solve the system: $y=-\sqrt{x}$
$(x-3)^2+y^2=5$

This is another one that should be solved by the substitution method since one of our variables is already solved for. We will substitute $-\sqrt{x}$ into the bottom equation for y: $(x-3)^2+(-\sqrt{x})^2=5$. So now the equation becomes: $(x-3)^2+x=5$. We will expand this: $x^2-6x+9+x=5$. We can add like terms: $x^2-5x+9=5$. We need to set this equal to zero: $x^2-5x+4=0$. Factoring this we get: $(x-1)(x-4)=0$. We get two answers for x: x = 1 and x = 4. We can put these into the first equation to get our corresponding answers for y. When x = 1 we have $y=-\sqrt{1}$, so y = -1. When x = 4 we have $y=-\sqrt{4}$ so y = -2. So our two points of intersection will be (1, -1) and (4, -2).




NOTE: Suppose you used the second equation to solve for y instead of the first equation. If x = 1 and you put this into the second equation you would have gotten $y=\pm 1$. You need to test this with the first equation. For example on point we got was (1, 1). If we test this with the first equation we would get $1=-\sqrt{1}$ which is not a true statement so (1, 1) is not a solution. Look at the graph. We should only have two points of intersection.

EXAMPLE: Solve the system: $\frac{5}{x^2}-\frac{2}{y^2}=-3$, $\frac{3}{x^2}+\frac{1}{y^2}=7$. Hint: Let $u=\frac{1}{x^2}$ and $v=\frac{1}{y^2}$

First we make the indicated substitutions to eliminate the fractions: $5u-2v=-3$, $3u+v=7$. I will now eliminate the *v*.

First I will multiply the last row by 2 to get: $5u-2v=-3$, $6u+2v=14$. Then add the equations together to get $11u = 11$.

So $u = 1$. Now we need to solve for v by putting it into either equation. I will use the top equation: $5(1)-2v=-3$. So $-2v=-8$, so $v=4$. So we are not done with this problem. The original problem has *x* and *y*, so that is now what we need to solve for. We will go back to our substitutions: $u=\frac{1}{x^2}$ so $1=\frac{1}{x^2}$, giving us $x^2=1$, and then $x=\pm 1$. Next, $4=\frac{1}{y^2}$, so $4y^2=1$. Then $y^2=\frac{1}{4}$ giving us $y=\pm\frac{1}{2}$. We will now write our answer as coordinates: (1, ½), (1, –½), (–1, ½), (–1, –½).