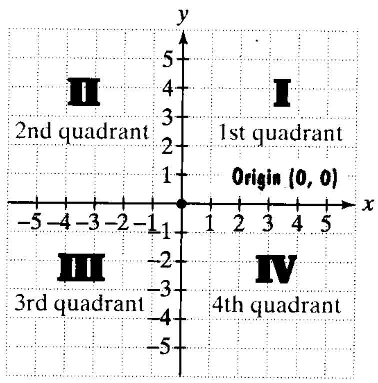
1.1 The Distance and Midpoint Formulas

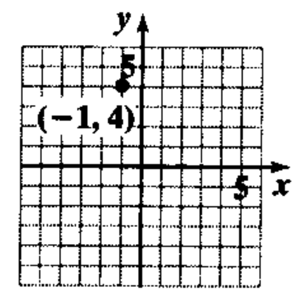
Cartesian Coordinate System – This is the standard graphing system we will use in this course for graphing all kinds of equations. The graph is made up of four quadrants as shown below:



The vertical axis is called the y axis and the horizontal axis is called the x-axis. The place where the two axes come together is called the origin and the coordinates are (0, 0). We write points in the form (x, y). A point is also referred to as an ordered pair. The first number x represents the horizontal change, and the second number y represents the vertical change. You move to the right for positive x values and to the left for negative x values. You move up for positive y values and down for negative y values.

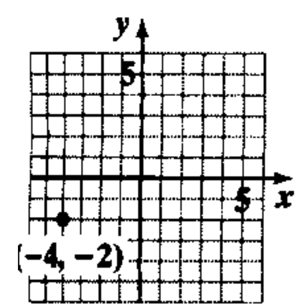
Plotting points To plot a point, start at (0, 0). Then move left or right depending on the x value and up or down depending on the y value.

EXAMPLE: Plot (-1, 4).



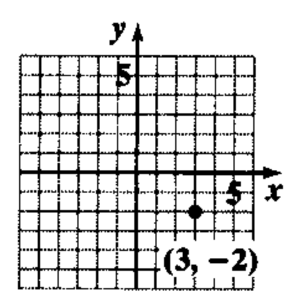
First we start at (0, 0). Since the x value is negative we will first move 1 place to the left. Then from this spot we will move up 4 places since the y-value is positive.

EXAMPLE: Plot (-4, -2).



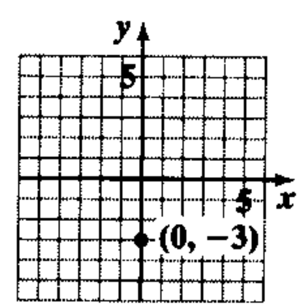
First we start at (0, 0). Since the x value is negative we will first move 4 places to the left. Then from this spot we will move down 2 places since the y-value is negative.

EXAMPLE: Plot (3, -2).



First we start at (0, 0). Since the x value is positive we will first move 3 places to the right. Then from this spot we will move down 2 places since the y-value is negative.

EXAMPLE: Plot (0, -3).

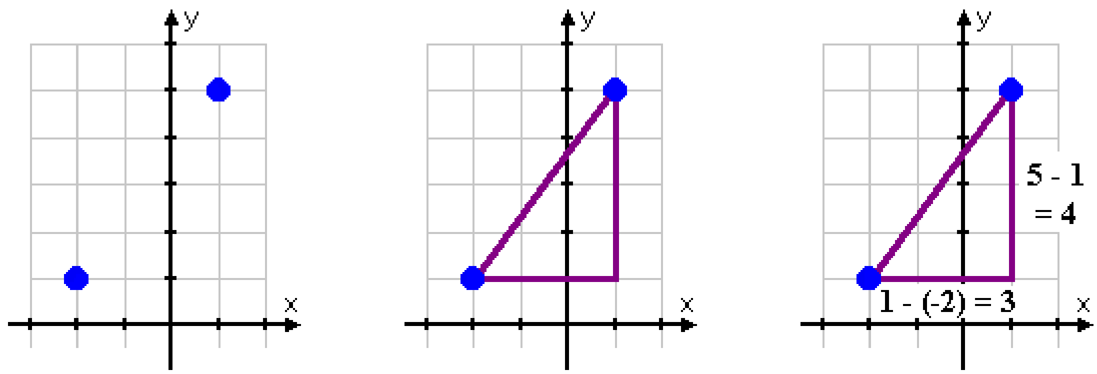


First we start at (0, 0). Since the x value is zero we will not move in either direction. We will stay on the y-axis. Then from this spot we will move down 3 places since the y-value is negative.

Distance Formula

The distance formula is used to find the distance between two points (x_1, y_1) and (x_2, y_2).

Let's first start with two points, (-2,1) and (1,5). First we plot the points. Then we will connect the points with a line. I will also darken the vertical and horizontal differences of the points. A right triangle is now formed. I will now label the actual vertical and horizontal distance.



Since we have a right triangle, we can use the Pythagorean Theorem to find the length of the longest side, which is our distance. The Pythagorean Theorem says that if you have a right triangle, then $a^2 + b^2 = c^2$ where c is the longest side of the triangle. So, we will solve the equation:

$(3)^2 + (4)^2 = c^2$

$9 + 16 = c^2$ We are not done. We need to solve for c, so take the square root of both sides.

$\pm 5 = c$ We get a positive and a negative answer, however distance is always positive. Therefore, the distance between these two points is 5 units.

There is an easier way to find the distance instead of plotting. In the drawing above, we see that we can find the length of the vertical side by subtracting the y-values. Generalized, we can say this represents $y_2 - y_1$. To find the horizontal side, we subtract the x-values, so generalizing we can say this represents $x_2 - x_1$. Using these algebraic expressions, we can solve for the diagonal of the triangle, which is our distance. Therefore:

If you have points (x_1, y_1) and (x_2, y_2) then:

the **distance formula** is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

EXAMPLE: Use the distance formula to find the distance between the points (-2, 4) and (1, 6)

It does not matter the order that the points are in. I will let the (-2, 4) be (x_1, y_1) and (1, 6) be (x_2, y_2). We will now substitute these numbers into the distance formula:

$d = \sqrt{(1-(-2))^2 + (6-4)^2}$

$d = \sqrt{3^2 + 2^2} = \sqrt{13}$

You don't need to turn this into a decimal.

There are different applications the distance formula can be used for. One example is circle equations, which we will look at in the next section.

Midpoint Formula

The midpoint is the halfway point on a line. If the line is formed by the points (x_1, y_1) and (x_2, y_2), then

The midpoint is: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Notice our answer is a point (x, y). This divides the line into two pieces of equal length.

EXAMPLE: Find the midpoint of a line segment containing (2, -3) and (4, 2).

I will let the (2, -3) be (x_1, y_1) and (4, 2) be (x_2, y_2). Plug these into the formula:

$$M = \left(\frac{2+4}{2}, \frac{-3+2}{2}\right)$$

$$M = \left(3,\ -\frac{1}{2}\right)$$ This is as far as we can simplify, so we have found the midpoint.

EXAMPLE: If the point (11,14) is shifted 5 units to the right and 2 units down, what are the new coordinates?

When you move the point to the right or left, this means you are working with the x-value.
When you move the point up and down, you are working with the y-value.

Moving to the right means you add a number to x-value, moving to the left is subtraction.
Moving up means you add a number to the y-value, moving down is subtraction.

Our answer is: (11 + 5, 14 – 2) = (16,12)