2.3 Properties of Functions

Increasing, Decreasing, Constant Graphs; Relative Extrema

First we will start with some definitions:

Increasing: as x increases, y increases (graph goes uphill as you move from left to right)

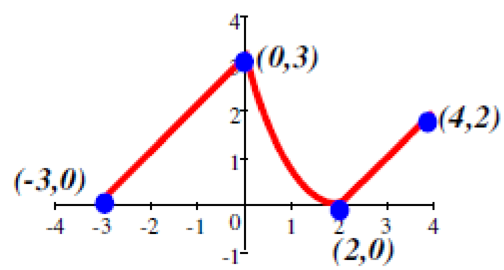
Decreasing: as x increases, y decreases (graph goes downhill as you move from left to right)

Constant: as x increases, y does not change (this part of the graph is horizontal)

Relative maximum: a point at which the graph increases and then decreases (peak)

Relative minimum: a point at which the graph decreases and then increases (valley)

EXAMPLE: Use the graph below to answer the following questions



a.) Indicate the interval(s) of which f is increasing There are two places this occurs. Between the x values of-3 and 0 the graph is climbing. This also happens between 2 and 4. We write the answer as $(-3,0)\cup(2,4)$. We always use parenthesis because at the endpoints the graph is not increasing or decreasing.

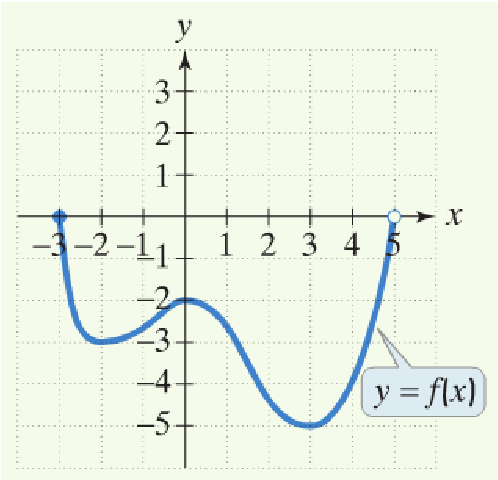
b.) Indicate the interval(s) of which f is decreasing. There is one place where the graph is falling as we move from left to right. This is between the x value of 0 and 2, so we write our answer as (0, 2).

c.) List the number where f has a relative maximum. This is asking for the x value at which the graph has a local maximum. This occurs at x = 0.

d.) What is the relative maximum?
This is asking for the y-value of the local max, which is 3.

e.) What is the relative minimum?
The y-value of the local minimum is 0.

EXAMPLE: Use the graph below to answer the following questions



a.) Indicate the interval(s) of which f is increasing
$(-2,0) \cup (3,5)$

b.) Indicate the interval(s) of which f is decreasing
$(-3,-2) \cup (0,3)$

c.) List the number(s) where f has a relative minimum.
This is asking for the x value(s) at which the graph has a local minimum. This occurs at x = -2 and at x = 3.

d.) What is the relative maximum(s)?
This is asking for the y-value of the local max, which is -2.

e.) What is the relative minimum(s)?
The y-value of the local minimum is -3 and -5.

f.) What is the domain?
[-3, 5)

g.) What is the range?
[-5, 0]

Even and Odd functions

If $f(-x) = f(x)$ then the function is even, and symmetric to the y-axis.
If $f(-x) = -f(x)$ then the function is odd, and symmetric to the origin.

EXAMPLE: Determine whether the following are even, odd, or neither.

a.) $f(x) = x^4 + 7$

We want to put a –x in for x first. You will get: $f(-x) = (-x)^4 + 7$ This simplifies to $f(-x) = x^4 + 7 = f(x)$. Since this is the same as the original, this function is even.

b.) $f(x) = 6x^5 - x^3$

We want to put a –x in for x first. You will get: $f(-x) = 6(-x)^5 - (-x)^3$ This simplifies to $f(-x) = -6x^5 + x^3$. Factoring out a negative: $f(-x) = -(6x^5 - x^3)$. So we have $f(-x) = -f(x)$ so this function is odd.

c.) $f(x) = x^2 + x$

We want to put a –x in for x first. You will get: $f(-x) = (-x)^2 + (-x)$ This simplifies to $f(-x) = x^2 - x$. There is nothing more I can do to this one. First we notice this is not the same as the original so it is definitely not even. If we try to factor out a negative we get $f(-x) = -(-x^2 + x)$. Since you don't have f(x) inside of the parenthesis it is not odd either. We will answer neither.

d.) $f(x) = \frac{|2x|}{x}$

If we put in a –x for x we will get: $f(-x) = \frac{|2(-x)|}{-x}$ which simplifies to $f(x) = -\frac{|2x|}{x} = -f(x)$ which means the function will be odd.

e.) $f(x) = \frac{x^3}{x^2 - 9}$

If we put in a –x for x we will get: $f(-x) = \frac{(-x)^3}{(-x)^2 - 9}$ which simplifies to $f(-x) = \frac{-x^3}{x^2 - 9} = -\frac{x^3}{x^2 - 9} = f(-x)$ which means the function will be odd.

Average Rate of Change (A.R.C.)

The A.R.C. is an estimate of the slope between x and c. Basically how much does something change between x and c. The formula is as follows and is derived from the slope formula.

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

EXAMPLE: Find the A.R.C. for the $f(x) = -3x^2 + 3$ from $x_1 = 0$ to $x_2 = 2$.

We can substitute these numbers into our general A.R.C. formula:

$\frac{f(2) - f(0)}{2 - 0}$ Now we will work this out. Find $f(2)$ and $f(0)$

$\frac{-9 - 3}{2} = -6$ So the slope is -6 between the x values of 0 and 2.

EXAMPLE: Find the A.R.C. for the $f(x) = x^3 - x + 2$ from $x_1 = 1$ to $x_2 = 3$.

We can substitute these numbers into our general A.R.C. formula:

$\frac{f(3) - f(1)}{3 - 1}$ Now we will work this out. Find $f(3)$ and $f(1)$

$\frac{26 - 2}{2} = 12$ So the slope is 12 between the x values of 1 and 3.

EXAMPLE: Find the A.R.C. for the $f(x) = \sqrt{x}$ from $x_1 = 9$ to $x_2 = 16$.

We can substitute these numbers into our general A.R.C. formula:

$\frac{f(16) - f(9)}{16 - 9}$ Now we will work this out. Find $f(16)$ and $f(9)$

$\frac{4 - 3}{7} = \frac{1}{7}$ So the slope is $\frac{1}{7}$ between the x values of 9 and 16.