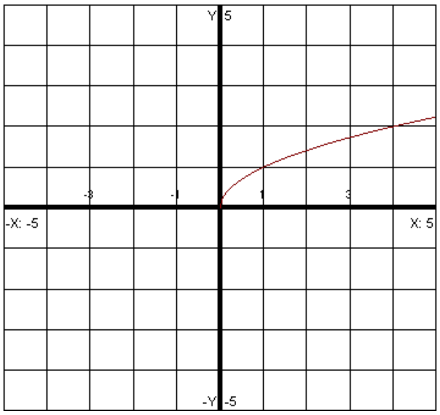
2.4 Library of Functions; Piecewise Functions

We will first look at a library of functions you should know how to sketch:



$y = \sqrt{x}$

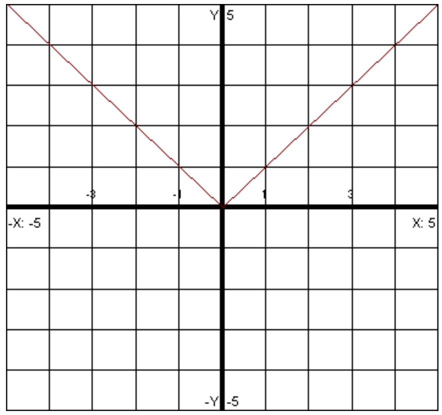
Square Root Function

Domain: $[0, \infty)$

Range: $[0, \infty)$

Increasing: $(0, \infty)$

Decreasing: None



$y = |x|$

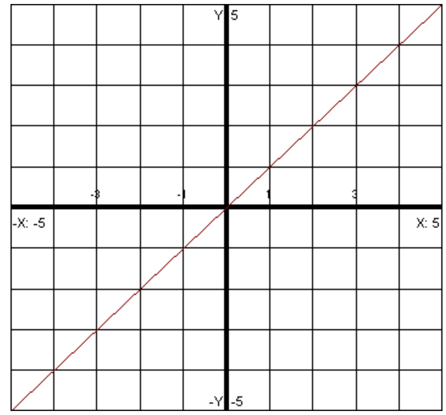
Absolute Value Function

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$



$y = x$

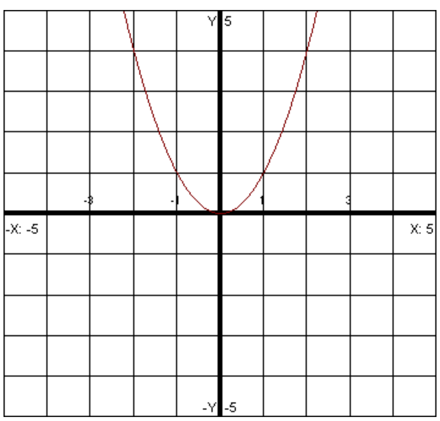
Identity Function

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing: $(-\infty, \infty)$

Decreasing: None



$y = x^2$

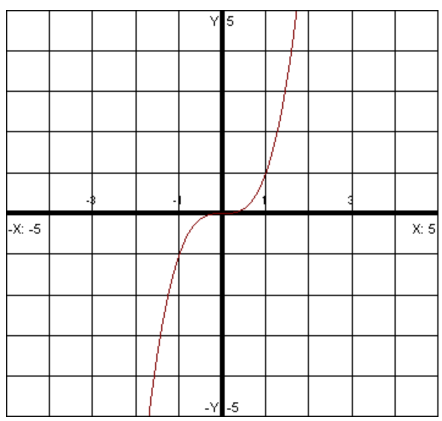
Standard Quadratic Function

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$



$y = x^3$

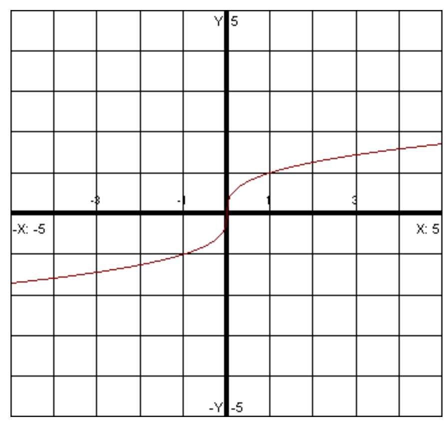
Standard Cube Function

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing: $(-\infty, \infty)$

Decreasing: None



$y = \sqrt[3]{x}$

Cube Root Function

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing: $(-\infty, \infty)$

Decreasing: None

Piecewise Functions

These functions are made up of different pieces. Each piece is defined for certain values of x.

EXAMPLE: Use the function $f(x)=\begin{cases} x+2 & if \quad x<-3 \\ x-2 & if \quad x\geq -3 \end{cases}$ to find $f(-4)$, $f(-3)$ and $f\left(-\frac{3}{2}\right)$. Then graph. and use this to determine the graph's range.

a.) $f(-4)$ In order to know which equation we are using, look at the number inside the parenthesis, which is -4. In our function, we need to find what function includes -4. This would be the first equation since -4 is less than -3. So we put -4 in for x in the first equation. You will get -4 + 2 = -2. So $f(-4)=-2$.

b.) $f(-3)$ The equation that includes -3 would be the second one since it is greater than or equal to -3. So we will place the -3 in for x in the second equation: -3 – 2 = -5. So $f(-3)=-5$.

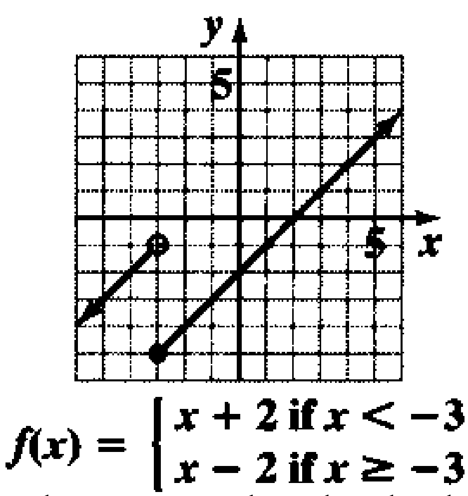
c.) $f\left(-\frac{3}{2}\right)$ If you don't know if this fraction is less or more than -3 then turn it into a decimal. This is -1.5.

So this would be greater than -3, so we will use the second equation. $-\frac{3}{2}-2=\frac{-7}{2}$. So $f\left(-\frac{3}{2}\right)=-\frac{7}{2}$.

Now we do a graph. Notice that there are two different equations we need to graph. To graph these, we will make a table of values for each one. The most important thing is that we need to plug in x values that match our conditions in the problem. For example the first equation says we need to use x values that are less than but not equal to -3. Now we can plug in -3, and this will end up as an open circle on the graph since it is not included. So we can use x = -5, -4, -3. Three points are enough. For the second equation we need to pick x values that are greater than or equal to -3. When we plug in -3 we can plot this point as a closed circle since this point is included. We will use x = -3, -2, -1. Below are a table of values for each equation. To graph this, we plot points from our table making sure to indicate a closed or open circle:

x	$y=x+2$	(x, y)
-5	$y=-5+2=-3$	(-5, -3)
-4	$y=-4+2=-2$	(-4, -2)
-3	$y=-3+2=-1$	(-3, -1)

x	$y=x-2$	(x, y)
-3	$y=-3-2=-5$	(-3, -5)
-2	$y=-2-2=-4$	(-2, -4)
-1	$y=-1-2=-3$	(-1, -3)



Notice the open circle at (-3, -1). Also notice that there are no x values plotted to the right of the open circle because this graph is only defined for values less than or equal to -3. Notice the closed circle at (-3, -5). Notice that there are not x values plotted to the left of this point since we are only allowed to use values for x that are greater than or equal to -3. The range is the y-values the graph is using. This would be ALL y values, so the answer is $(-\infty,\ \infty)$.

EXAMPLE: Use the function $f(x)=\begin{cases}-\frac{1}{2}x^2 & if \quad x<1 \\ 2x+1 & if \quad x\geq 1\end{cases}$ to find $f(-1)$, $f(1)$ and $f\left(\sqrt{\frac{9}{10}}\right)$. Then graph. and use this to determine the graph's range.

a.) $f(-1)$ In order to know which equation we are using, look at the number inside the parenthesis, which is -1. In our function, we need to find what function includes -1. This would be the first equation since -1 is less than 1. So we put -1 in for x in the first equation. You will get $-\frac{1}{2}(-1)^2=-\frac{1}{2}(1)=-\frac{1}{2}$. So $f(-1)=-\frac{1}{2}$.

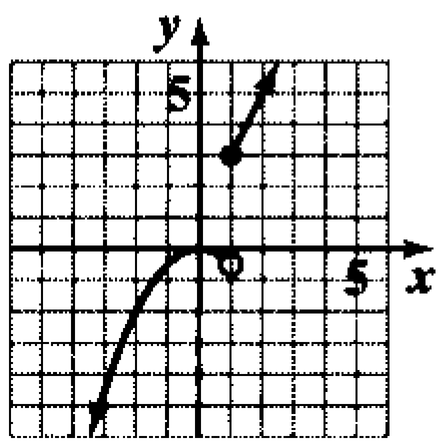
b.) $f(1)$ The equation that includes 1 would be the second one since it is greater than or equal to 1. So we will place the 1 in for x in the second equation: 2(1) + 1 = 2 + 1 = 3. So $f(1)=3$.

c.) $f\left(\sqrt{\frac{9}{10}}\right)$ If you don't know if this is less than 1 then turn it into a decimal. This would be 0.95, which is than 1, so we will use the first equation. $-\frac{1}{2}\left(\sqrt{\frac{9}{10}}\right)^2=-\frac{1}{2}\left(\frac{9}{10}\right)=-\frac{9}{20}$. So, $f\left(\sqrt{\frac{9}{10}}\right)=-\frac{9}{20}$.

Now we do a graph. Notice that there are two different equations we need to graph. To graph these, we will make a table of values for each one. The most important thing is that we need to plug in x values that match our conditions in the problem. For example the first equation says we need to use x values that are less than but not equal to 1. Now we can plug in 1, and this will end up as an open circle on the graph since it is not included. So we can use x = -1, 0, 1. Three points are enough. For the second equation we need to pick x values that are greater than or equal to 1. When we plug in 1 we can plot this point as a closed circle since this point is included. We will use x = 1, 2, 3. Below are a table of values for each equation. To graph this, we plot points from our table making sure to indicate a closed or open circle:

x	$y=-\frac{1}{2}x^2$	(x, y)
-1	$y=-\frac{1}{2}(-1)^2=-\frac{1}{2}(1)=-\frac{1}{2}$	$\left(-1,\ -\frac{1}{2}\right)$
0	$y=-\frac{1}{2}(0)^2=0$	(0, 0)
1	$y=-\frac{1}{2}(1)^2=-\frac{1}{2}$	$\left(1,\ -\frac{1}{2}\right)$

x	$y=2x+1$	(x, y)
1	$y=2(1)+1=3$	(1, 3)
2	$y=2(2)+1=5$	(2, 5)
3	$y=2(3)+1=7$	(3, 7)



$$f(x)=\begin{cases}-\frac{1}{2}x^2 \text{ if } x<1 \\ 2x+1 \text{ if } x\geq 1\end{cases}$$

The range is the y-values the graph is using. The graph is not using any y value between 0 and 3, so we write our answer this way: $(-\infty,\ 0]\cup[3,\ \infty)$.

EXAMPLE: Use the function $f(x)=\begin{cases} 0 & if \quad x<-3 \\ -x & if \quad -3\le x<0 \\ x^2-1 & if \quad x\ge 0 \end{cases}$ to find $f(0)$, $f(-1)$, $f\left(\frac{\sqrt{10}}{2}\right)$, $f(-\sqrt{11})$.

a.) $f(0)$ Zero is only included in the third equation. We put a zero in for x in the third equation: $0^2-1=-1$. So we write $f(0)=-1$

b.) $f(-1)$ Negative one is included in the middle equation since it is between -3 and 0. We will put in -1 for x. We will get: $-(-1)=1$. So we write $f(-1)=1$.

c.) $f\left(\frac{\sqrt{10}}{2}\right)$ Turning it into a decimal we get 1.58, so we use the third equation this time. So

$$f\left(\frac{\sqrt{10}}{2}\right)=\left(\frac{\sqrt{10}}{2}\right)^2-1 = \frac{10}{4}-1 = \frac{5}{2}-1 = \frac{3}{2}.$$

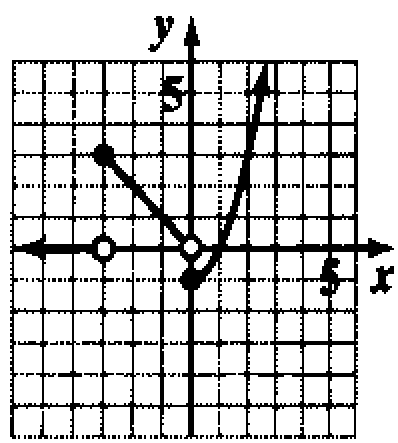
d.) $f(-\sqrt{11})$ Turning this into a decimal we get -3.32, so we use the first equation. The answer is 0.

Now we do a graph by making a table of values for each equation. Notice that there are three different equations we need to graph. To graph these, we will make a table of values for each one. For the first equation we use the following x values: -5, -4, -3. For the second equation we can use x = -3, -1, 0. For the third equation we can use x = 0, 1, 2.

x	$y=0$	(x, y)
-5	$y=0$	(-5, 0)
-4	$y=0$	(-4, 0)
-3	$y=0$	(-3, 0)

x	$y=-x$	(x, y)
-3	$y=-(-3)=3$	(-3, 3)
-1	$y=-(-1)=1$	(-2, 2)
0	$y=-(0)=0$	(0, 0)

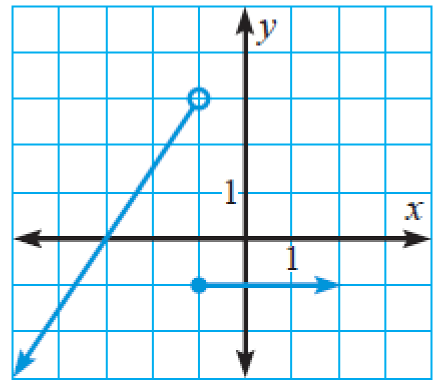
x	$y=x^2-1$	(x, y)
0	$y=(0)^2-1=-1$	(0, -1)
1	$y=(1)^2-1=0$	(1, 0)
2	$y=(2)^2-1=3$	(2, 3)



$$f(x)=\begin{cases} 0 & \text{if } x<-3 \\ -x & \text{if } -3\le x<0 \\ x^2-1 & \text{if } x\ge 0 \end{cases}$$

Notice that our first equation is a horizontal line with an open circle at (-3, 0). Notice an open circle at (0, 0), and closed circle at (0, -1). The range is the y-values the graph is using. The graph is not using any y values less than -1, so we write our answer as: $[-1, \infty)$.

EXAMPLE: Write equations for the piecewise function whose graph is shown below:



First let's look at the line on the left. Since this is a line, we will write it in the form y = mx + b. We can get the slope from line. It is rise/run = $m = 3/2$. We also see that the x-intercept is –3. So $x = -3$ and $y = 0$. Let's put this information into our line and solve for b: $0 = 3/2(-3) + b$. So $0 = -9/2 + b$. Solving gives us $b = 9/2$. Therefore the equation for the left line is $y = (3/2)x + 9/2$. The other line is horizontal and crosses the y-axis at –1. Therefore this equation is $y = -1$. We see that these lines start or stop at the x value of –1. So that is our constraint. We are now ready to write our piecewise function: $f(x) = \begin{cases} \frac{3x}{2} + \frac{9}{2} & if \quad x < -1 \\ -1 & if \quad x \geq -1 \end{cases}$