2.5 Transformations of Functions

Transformations and Graph Sketches

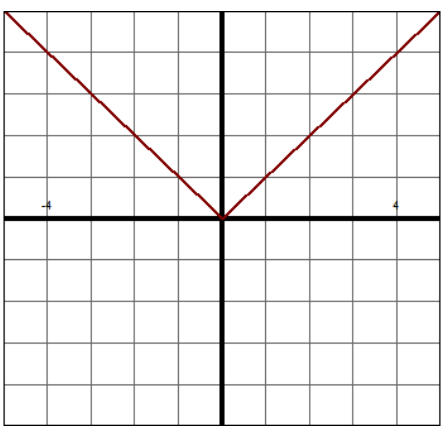
When we used to graph a line the usual thing to do was make a table of values and plot the points. This method works but takes a long time. Transformations allows you move a graph up or down, left or right into a new position. We start with the basic graphs we learned in the last section and will move it based on the following criteria.

Suppose $y = f(x)$ is the original function (one we looked at in a previous section)

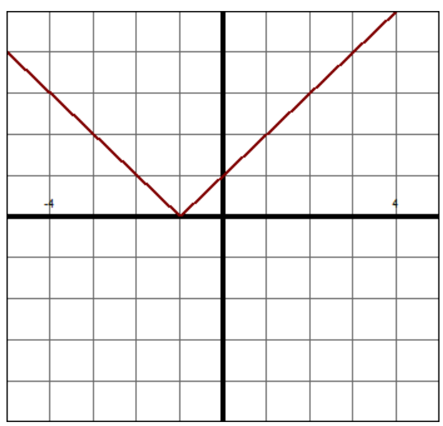
$y = f(x) + k$ moves $f(x)$ k units up
$y = f(x) - k$ moves $f(x)$ k units down
$y = f(x - h)$ moves $f(x)$ h units to the right
$y = f(x + h)$ moves $f(x)$ h units to the left
$y = -f(x)$ flips the graph over the horizontal axis
$y = f(-x)$ flips the graph over the vertical axis
$y = a \cdot f(x)$ If $|a|>1$ then there is a vertical stretch. If $0 < |a| < 1$, then there is a vertical compression.

Let's look at some examples. For all of these we are just making a sketch of the function.

EXAMPLE: Sketch $y = |x+1|$ by using transformations.

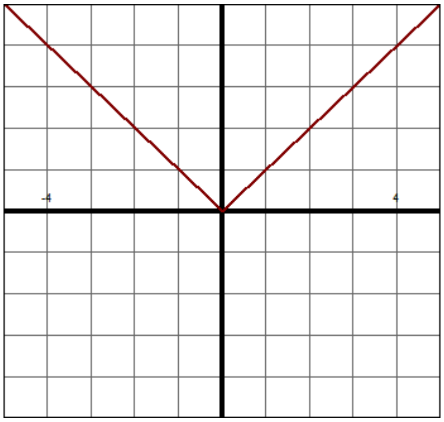


First start with the correct base graph of $y = |x|$.

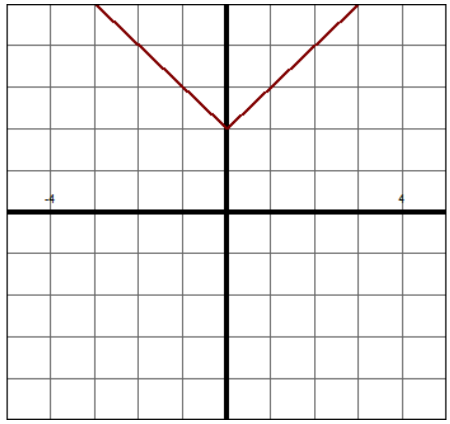


We notice the $x + 1$ inside of the absolute value. This means we move the graph of $y = |x|$ one unit to the left because of our transformation rules.

EXAMPLE: Sketch $y = |x| + 2$ by using transformations.

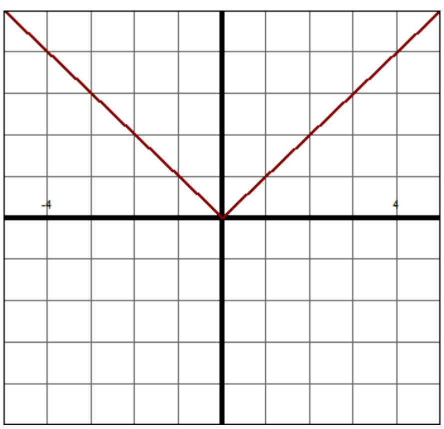


First start with the correct base graph of $y = |x|$.

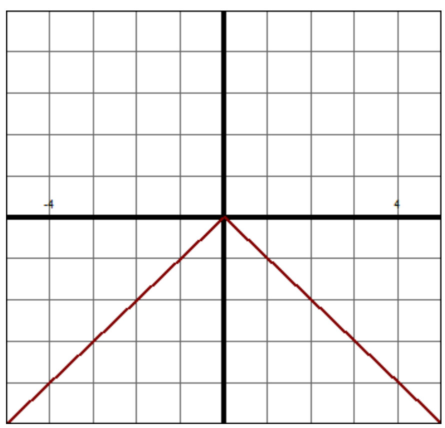


We notice the +2 outside of the absolute value. This means we move the graph of $y = |x|$ two units up because of our transformation rules.

EXAMPLE: Sketch $y = -|x|$ by using transformations.



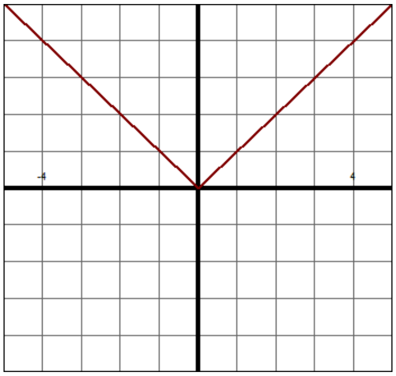
First start with the correct base graph of $y = |x|$.



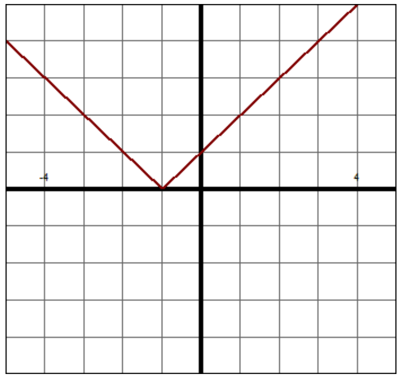
We notice the negative outside of the absolute value. This means we flip the graph of $y = |x|$ over the horizontal axis because of our transformation rules.

Now let's combine some transformations that we previously looked at separately.

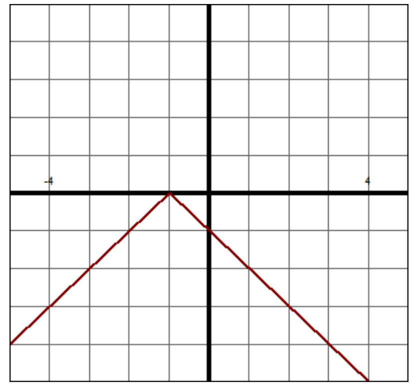
EXAMPLE: Sketch $y = -|x+1| + 2$ by using transformations.



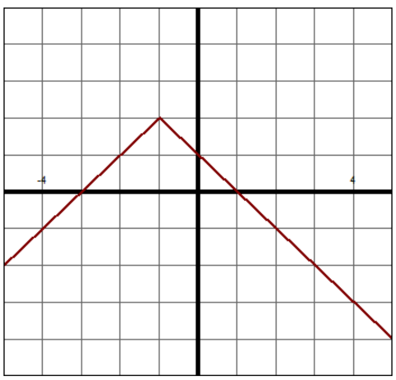
First start with the correct base graph of $y = |x|$.



To graph $y = |x+1|$ we will move the graph $y = |x|$ one unit to the left.



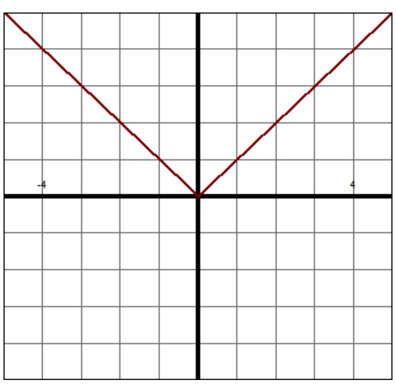
Next we will graph $y = -|x+1|$
To do this we will flip the graph $y = |x+1|$ over the vertical axis.



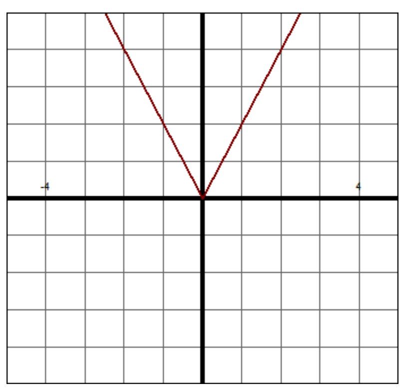
Finally we will move $y = -|x+1|$ up 2 units. We now have the graph $y = -|x+1| + 2$, which is our answer.

EXAMPLE: Sketch $y = |2x|$ by using transformations.

First let's simplify. We can rewrite this as $y = 2|x|$. This means we will have a vertical stretch.

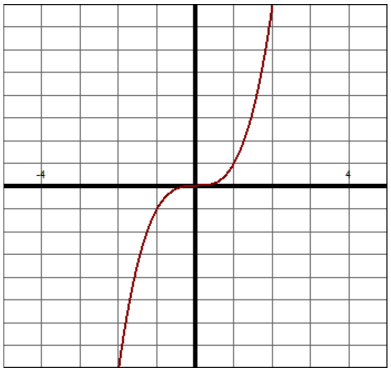


First start with the correct base graph of $y = |x|$.

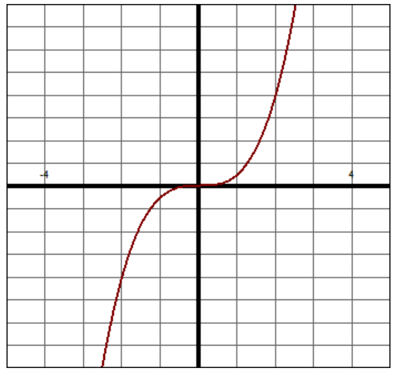


Now we will double all the y-values.
For example, a point on $y = |x|$ was (1, 1).
So the same point on $y = 2|x|$ is (1, 2).

EXAMPLE: Sketch $y = \frac{1}{2}x^3$ by using transformations.

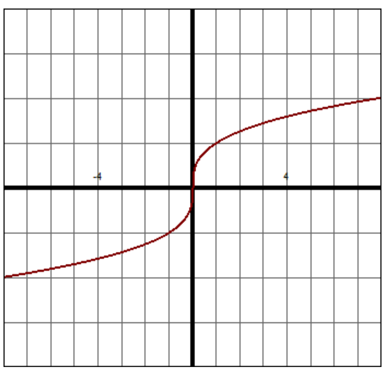


First start with the correct base graph of $y = x^3$.

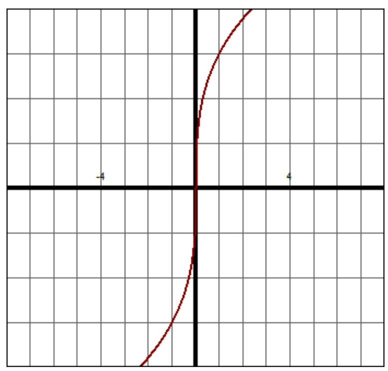


Now we will multiply all the y-values by 1/2. So a point originally at (2, 8) is now at (2, 4).

EXAMPLE: Sketch $y = 3 \cdot \sqrt[3]{x}$ by using transformations.

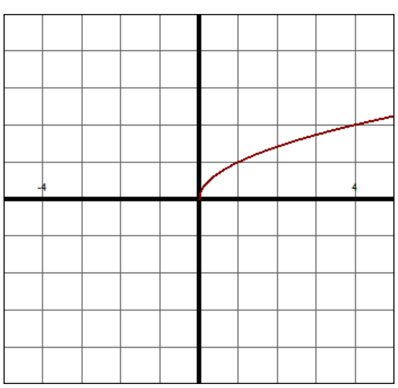


First start with the correct base graph of $y = \sqrt[3]{x}$.

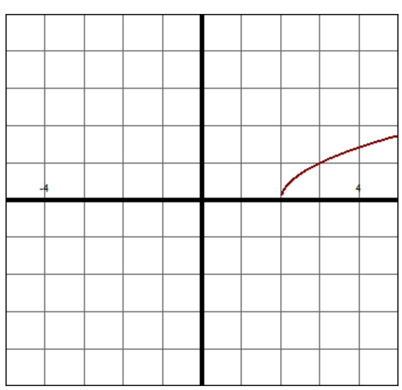


Now we will multiply all the y-values by 3. So a point originally at (1, 1) is now at (1, 3).

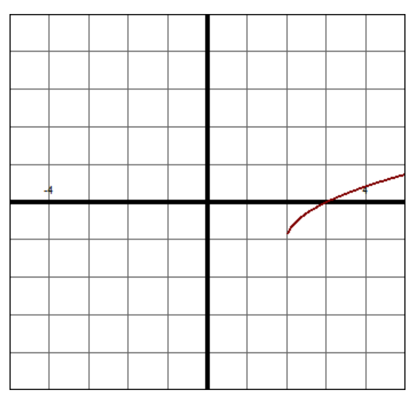
EXAMPLE: Sketch $y = \sqrt{x-2} - 1$ by using transformations.



First start with the correct base graph of $y = \sqrt{x}$.

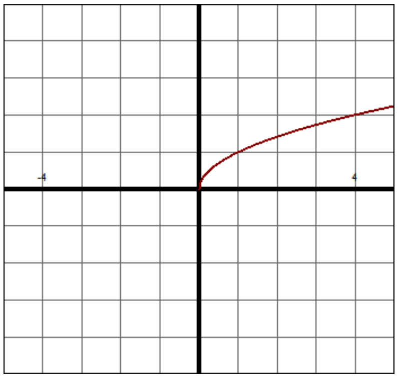


To graph $y = \sqrt{x-2}$ we will move the graph $y = \sqrt{x}$ two units to the right.

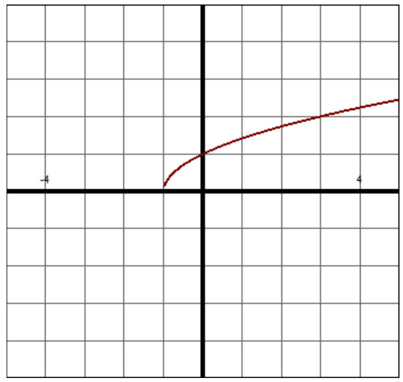


Now we will move the graph $y = \sqrt{x-2}$ down one unit. This is our final answer.

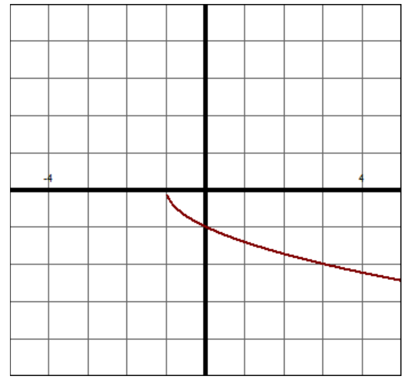
EXAMPLE: Sketch $y = -\sqrt{x+1} - 2$ by using transformations.



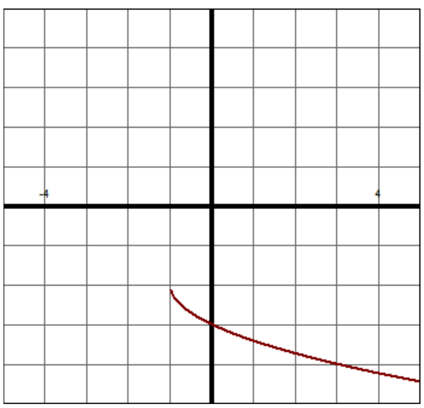
First start with the correct base graph of $y = \sqrt{x}$.



To graph $y = \sqrt{x+1}$ we will move the graph $y = \sqrt{x}$ one unit to the left.



To graph $y = -\sqrt{x+1}$ we will flip $y = \sqrt{x+1}$ over the horizontal axis.



Now we move the graph $y = -\sqrt{x+1}$ down two units.
This graph is our final answer, which is $y = -\sqrt{x+1} - 2$.

More on next page…

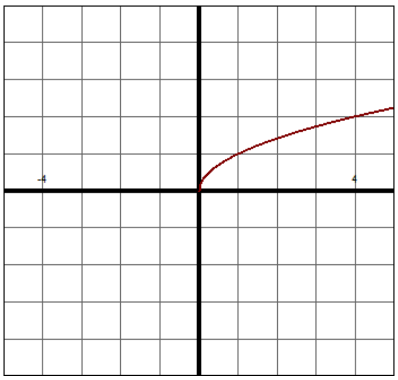
EXAMPLE: Sketch $y = \sqrt{4 - x} + 2$ by using transformations.

In order to use the transformation rules the *x* must come first and there must be a one in front of *x*. In our problem above we need to first put the x first and then we will factor out a negative:

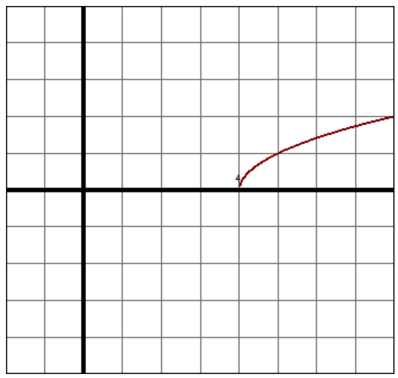
$y = \sqrt{4 - x} + 2$

$y = \sqrt{-x + 4} + 2$ Here we put the *x* term first

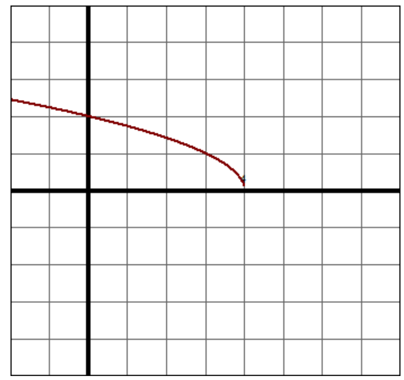
$y = \sqrt{-(x - 4)} + 2$ Here we factored out a -1. Now we will graph it.



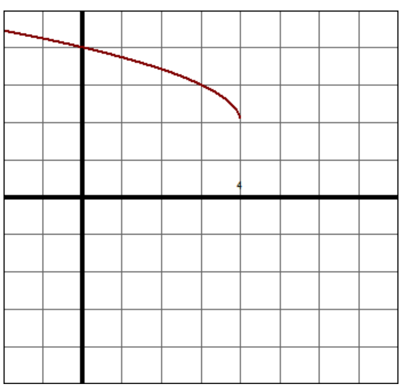
First start with the correct base graph of $y = \sqrt{x}$.



To graph $y = \sqrt{x - 4}$ we will move the graph $y = \sqrt{x}$ four units to the right.



To graph $y = \sqrt{-(x - 4)}$ we will flip $y = \sqrt{x - 4}$ over the vertical axis.

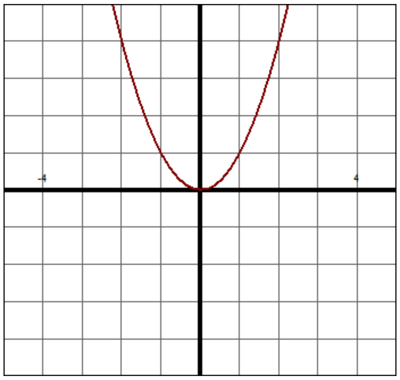


Now we will move the graph $y = \sqrt{-(x - 4)}$ up two units.

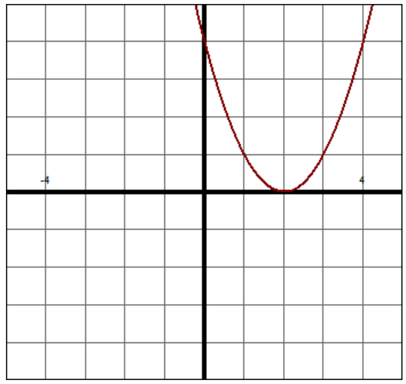
This graph is our final answer, which is $y = \sqrt{4 - x} + 2$.

More on next page…

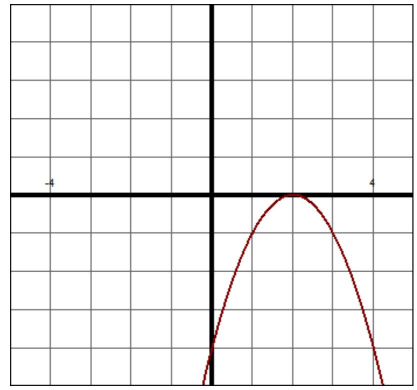
EXAMPLE: Sketch $y = -\frac{1}{2}(x-2)^2 - 1$ by using transformations.



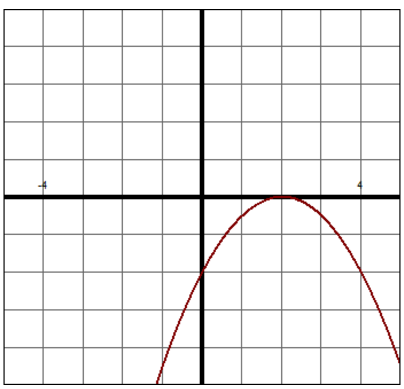
First start with the correct base graph of $y = x^2$.



To graph $y = (x-2)^2$ we will move the graph $y = x^2$ two units to the right.



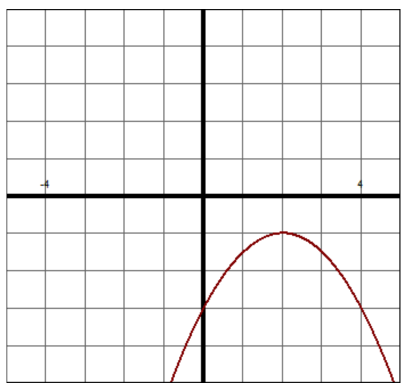
To graph $y = -(x-2)^2$ we will flip $y = (x-2)^2$ over the horizontal axis.



To graph $y = -\frac{1}{2}(x-2)^2$ we

need to cut all the y values in half.

This is a horizontal stretch.
For example, a point on $y = -(x-2)^2$ was (0, –4). This same point on $y = -\frac{1}{2}(x-2)^2$ is (0, –2).



To graph $y = -\frac{1}{2}(x-2)^2 - 1$

we will move the graph $y = -\frac{1}{2}(x-2)^2$

down one unit. This is our final answer.