2.6 Mathematical Models: Building Functions

EXAMPLE: A manufacturer buys a new machine costing $120,000. It is estimated that the machine has a useful lifetime of 10 years, and a salvage value of $4000 at that time. Find a linear function for the value, V, of the machine after t years.

Since it is linear, we know we will have a line. In order to get the equation of the line we need a slope and a point. We don't have a slope here, but we can find it by setting up two points and then using the slope formula. The time is going to represent x since and value is the y. This is because the value depends on the time, just like y depends on x. If the machine is new, then t = 0. At that time the value is $120,000 since it is new. The point is (0, 120000). We need another point. We are told that after 10 years the machine is worth $4000. Our point is (10, 4000). Now we have our two points so we can use the slope formula

$m = \dfrac{120000 - 4000}{0 - 10} = \dfrac{116000}{-10} = -11600$. This means that the machine's value drops by $11600 each year.

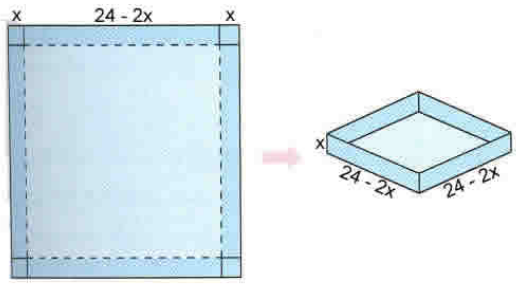
Now to find the equation we will use y = mx + b. We know m but we need x and y. Just use either point that we started with. I will use (10, 4000):

$y = mx + b$
$4000 = (-11600)(10) + b$
$b = 120000$
If we put these together into the equation we will get: $V(t) = -11600t + 120000$ which is the answer.

EXAMPLE: A company is planning to manufacture a certain product. The fixed costs will be $500000 and it will cost $400 to produce each product. Each will be sold for $600. What is the profit equation and how many units must be sold for in order to break even?

Profit is defined as the revenue minus the costs. We need to find our revenue and cost equations. The costs involve a fixed price plus a variable price. The equation is C = 400x + 500000. Since each is sold for $600 then this is the revenue, which is price times quantity. You will get R = 600x. To get the profit function you need to subtract the cost from the revenue: P = 600x – (400x + 500000). Simplifying you get: P = 200x – 500000. When you break even the profit will be zero. Put a zero in for P and solve for x: 0 = 200x – 500000. Solving this you will get x = 2500 units.

EXAMPLE: An open box of maximum volume is to be made from a square piece of material, 24 inches on a side, by cutting equal squares from the corners and turning up the sides (see figure). Write an equation for the volume, V, of the box as a function of x. Then find the volume when a 3-inch square is cut out.



To get the equation we notice that the height of this box is x. The length and width are both 24 – 2x. The formula for volume is V = LWH. So we have $V(x) = (24 - 2x)(24 - 2x)(x)$, or $V(x) = x(24 - 2x)^2$.

The volume when a 3-inch square is cut out is found by plugging in a 3 for x: $V(3) = 3(24 - 2(3))^2 = 3(18)^2 = 972$ cubic inches.

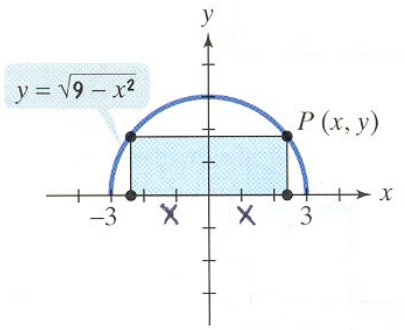
EXAMPLE: A wire of length $10x$ is bent into the shape of a circle. (a) Express the circumference of the circle as a function of x. (b) Express the area of the circle as a function of x.

(a) The circumference of a circle is the distance all the way around. Since the entire wire will be bent into the circle, this means that the distance all the way around will be $10x$. Therefore, $C(x) = 10x$. Make sure you do not put $C = 10x$. This is incorrect because it is not using the correct function notation.

(b) The area of a circle is $A = \pi r^2$. We just found that the circumference is $10x$. The formula for the circumference of a circle is $C = 2\pi r$. This means that $10x = 2\pi r$. We can now solve for the radius by dividing both sides by 2π: $\frac{10x}{2\pi} = \frac{2\pi r}{2\pi}$. Solving will give us $r = \frac{5x}{\pi}$. Now we will plug this into the area formula in place of r: $A = \pi\left(\frac{5x}{\pi}\right)^2 = \pi\left(\frac{25x^2}{\pi^2}\right) = \frac{25x^2}{\pi}$.

For our answer, we will write $A(x) = \frac{25x^2}{\pi}$.

EXAMPLE: A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{9 - x^2}$ (see figure). Express the area A of the rectangle as a function of x.



First we need to find the length and width of the rectangle. The length is going to be $2x$ since there are two x's in the figure. The height of the box is y, so $A = LW = 2xy$. Now we can plug in $\sqrt{9 - x^2}$ for y. So now the area formula is: $A(x) = 2x\sqrt{9 - x^2}$.