5.6 Exponential and Logarithmic Equations

In this section we will look at how to solve equations involving logarithms or exponents.

EXAMPLE: Solve: $\log_2 3x = 4$.

What need change it into exponential form and solve. Changing into exponential form we will get $2^4 = 3x$, which is $16 = 3x$. Divide and we will get $x = \frac{16}{3}$.

EXAMPLE: Solve: $\log_5 4x + 9 = 2$.

What need change it into exponential form and solve. Changing into exponential form we will get $5^2 = 4x + 9$, which is $25 = 4x + 9$. So, $16 = 4x$ and $x = 4$.

EXAMPLE: Solve: $2 \cdot \log_7 x = \log_7 16$

First we want to use property #5 from the previous section to bring up the 2. You will get: $\log_7 x^2 = \log_7 16$. Now we need to get rid of the logs. We can do this be changing from the log form to exponential. It doesn't matter which log you start with. I will change the log on the left side of the equation. Changing from log form to exponential you will get: $7^{\log_7 16} = x^2$. This simplifies to $16 = x^2$, so by solving this we get x = 4 and x = -4. What happens if we put x = -4 back into the original equation? That's right, you will get an error. The answer of x = -4 is not in the domain, so we need to discard it. So our only answer to this one is x = 4.

<u>MAKE SURE YOU ALWAYS CHECK YOUR ANSWERS TO SEE IF IT FITS THE DOMAIN!! IF NOT, YOU WILL NEED TO REJECT THIS ANSWER.</u>

EXAMPLE: Solve: $\log_8 (x + 7) = 1 - \log_8 (2 - x)$

The strategy with these problems is to first get all the logs on one side of the equation. Next the logs will be combined into one by using the log properties. Last we will change it into exponential form. Let's first get all the logs on one side of the equation: $\log_8 (x + 7) + \log_8 (2 - x) = 1$. Now we can use property #6 to combine these into a single log: $\log_8 (x + 7)(2 - x) = 1$. Now we need to change this into exponential form: $(x + 7)(2 - x) = 8^1$. We need to multiply on the left side of the equation: $-x^2 - 5x + 14 = 8$. Now set this equal to zero: $-x^2 - 5x + 6 = 0$. Now multiply both sides by -1 to get: $x^2 + 5x - 6 = 0$. Now factor: $(x + 6)(x - 1) = 0$. We get x = -6 and x = 1 as our answers. Now let's check to make sure both of these are in our domain. Put -6 into the original equation to get $\log_8 (-6 + 7) = 1 - \log_8 (2 - -6)$. We get: $\log_8 (1) = 1 - \log_8 (8)$. Since both of the numbers inside the log are positive, that means it is in our domain. Now let's try x = 1: $\log_8 (1 + 7) = 1 - \log_8 (2 - 1)$. You will get $\log_8 (8) = 1 - \log_8 (1)$. The numbers inside the log are positive, so we know that x = 1 is in our domain. Our answers are x = -6 and x = 1.

EXAMPLE: Solve: $\log_2(x+11)+\log_2(x+7)=5$

$\log_2(x+11)(x+7)=5$	First combine into one log.
$2^5=(x+11)(x+7)$	Change into exponential form
$32=x^2+18x+77$	Multiply and simplify
$0=x^2+18x+45$	Set it equal to zero
$0=(x+3)(x+15)$	Factor
$x=-3,\ \ x=-15$	These are our answers. Now we need to make sure they are in domain.

If we put -3 into the original we get $\log_2(-3+11)+\log_2(-3+7)=5$ which is $\log_2 8+\log_2 4=5$ This is okay since both 8 and 4 are in the domain. If we put in -15 we get $\log_2(-15+11)+\log_2(-15+7)=5$ which results in $\log_2(-4)+\log_2(-8)=5$. We can't have negative numbers inside a log, therefore -15 is not one of our answers. Our only answer for this problem is $x=-3$.

EXAMPLE: Solve: $\log_2(x+3)-\log_2(x+5)=1$

$\log_2(x+3)-\log_2(x+5)=1$	First combine into one log. This time we will turn it into a fraction
$\log_2\left(\frac{x+3}{x+5}\right)=1$	We used property #7. Now change into exponential form.
$\frac{x+3}{x+5}=2^1$	We can solve this by cross multiplying.
$2(x+5)=x+3$	Now solve for x.
$2x+10=x+3$	
$x=-7$	

If we put -7 into the original we get $\log_2(-7+3)-\log_2(-7+5)=1$ which is $\log_2(-4)+\log_2(-2)=1$ We can't have a negative inside the log, so we reject the answer x = -7. Since our only answer did not work, the answer to the problem is “no solution” or undefined.

EXAMPLE: Solve: $\log_3(x+1)=2+\log_3(2x-1)$

$\log_3(x+1)-\log_3(2x-1)=2$	Get all the logs on one side of the equation.
$\log_3\left(\frac{x+1}{2x-1}\right)=2$	First combine into one log. This time we will turn it into a fraction
$\frac{x+1}{2x-1}=3^2$	We used property #7. Now change into exponential form.
$\frac{x+1}{2x-1}=9$	We can solve this by cross multiplying.
$9(2x-1)=x+1$	Now solve for x.

$18x - 9 = x + 1$

$x = \frac{10}{17} \approx .59$ If we put this into the original problem we get $\log_3(.59+1) = 2 + \log_3(2(.59)-1)$.

There are positive number inside the logs, so x = 10/17 is our answer.

EXAMPLE: Solve: $\log_2(x-3) + \log_2 x - \log_2(x+2) = 2$

$\log_2\left(\frac{x(x-3)}{x+2}\right) = 2$ Combine into one log. Positive logs on top, negative logs on bottom.

$\frac{x(x-3)}{x+2} = 2^2$ We changed from log form into exponential form.

$\frac{x^2-3x}{x+2} = 4$ Now cross multiply.

$x^2 - 3x = 4(x+2)$ Now solve for x. Because it's a quadratic, we need to set it equal to zero.

$x^2 - 3x = 4x + 8$

$x^2 - 7x - 8 = 0$ Now factor and set each factor equal to zero.

$(x+1)(x-8) = 0$

x = -1, 8 If we put these into the original problem, we find that the only answer is x = 8.

EXAMPLE: Solve: $3^x = 7$.

For this problem, we can't use the Equal Bases Property since one base can't be written in terms of the other. To solve this we will take either the natural log or regular log of both sides:

$\ln 3^x = \ln 7$ Now use property #5 to bring down the x.

$x \ln 3 = \ln 7$ Divide by sides by $\ln 3$ to solve for x.

$x = \frac{\ln 7}{\ln 3}$. Note, you answer could also have been $x = \frac{\log 7}{\log 3}$, which is the same answer.

EXAMPLE: Solve: $2^{x+1} = 5^{1-2x}$.

$\ln 2^{x+1} = \ln 5^{1-2x}$ First take the natural log of both sides and use property #5 to bring down powers.

$(x+1)\ln 2 = (1-2x)\ln 5$ Now distribute.

$x\ln 2 + \ln 2 = \ln 5 - 2x\ln 5$ Get all the terms with x in it on one side of the equation.

$x\ln 2 + 2x\ln 5 = \ln 5 - \ln 2$ Now factor out an x.

$x(\ln 2 + 2\ln 5) = \ln 5 - \ln 2$ Solve for x.

$x = \frac{\ln 5 - \ln 2}{\ln 2 + 2\ln 5}$ We can leave our answer in this form. You do not need a decimal unless asked.

Are you allowed to cancel the ln 2 from top and bottom? The answer is NO. You can only do this if all the terms on the top and bottom are being multiplied. Therefore this is as far as we can go with simplifying.

EXAMPLE: Solve: $3^{2x} - 3^{x} - 72 = 0$.

This one will factor. In order to do this, let's make a substitution. We will let $u = 3^{x}$ and $u^{2} = 3^{2x}$. The always the middle term, ignoring the sign in front of it, so u is 3^{x}.

$u^{2} - u - 72 = 0$ We have substituted. Now it is easier to factor.

$(u-9)(u+8) = 0$ Now put back the u, which is $u = 3^{x}$.

$(3^{x} - 9)(3^{x} + 8) = 0$ Set each one equal to zero and solve each one individually.

$3^{x} - 9 = 0$

$3^{x} = 9$

$3^{x} = 3^{2}$ so x = 2.

$3^{x} + 8 = 0$

$3^{x} = -8$

$\ln 3^{x} = \ln(-8)$ The problem with this one is that -8 is not in our domain, so this is not one of our answers. The only answer is x = 2.

EXAMPLE: Solve: $2^{2x} - 7 \cdot 2^{x} + 12 = 0$.

This one will factor. In order to do this, let's make a substitution. We will let $u = 2^{x}$ and $u^{2} = 2^{2x}$. The always the middle term, ignoring the sign in front of it, so u is 3^{x}.

$u^{2} - 7u + 12 = 0$ We have substituted. Now it is easier to factor.

$(u-3)(u-4) = 0$ Now put back the u, which is $u = 2^{x}$.

$(2^{x} - 3)(2^{x} - 4) = 0$ Set each one equal to zero and solve each one individually.

$2^{x} - 3 = 0$

$2^{x} = 3$

$\ln 2^{x} = \ln 3$

$2^{x} - 4 = 0$

$2^{x} = 4$

Solving we get x = 2.

So $x \ln 2 = \ln 3$, and after dividing we get $x = \frac{\ln 3}{\ln 2}$. So our two answers are $x = 2$, $x = \frac{\ln 3}{\ln 2}$